1rBAT - 310114

1) a) $\dfrac{\sqrt{5}}{\sqrt{5}-\sqrt{3}} = \dfrac{\sqrt{5}(\sqrt{5}+\sqrt{3})}{(\sqrt{5}-\sqrt{3})(\sqrt{5}+\sqrt{3})} = \dfrac{5+\sqrt{15}}{5-3} = \boxed{\dfrac{5+\sqrt{15}}{2}}$

$\dfrac{\sqrt[3]{ab^5}}{\sqrt{a\sqrt{b^5}}} = \sqrt[12]{\dfrac{a^4b^{20}}{a^6b^{15}}} = \sqrt[12]{\dfrac{b^5}{a^2}} \cdot \sqrt[12]{\dfrac{a^{10}}{a^{10}}} = \boxed{\dfrac{\sqrt[12]{b^5a^{10}}}{a}}$

b) $\sqrt{5} + \sqrt{3125} = \sqrt{5} + \sqrt{5^5} = \sqrt{5} + 5^2\sqrt{5} = 26\sqrt{5} =$

$= \sqrt{26^2 \cdot 5} = \sqrt{3380} \Rightarrow \boxed{x = 3380}$

2) a) $3 - \dfrac{5x-2}{75} = \dfrac{7x}{30} \overset{(\cdot 150)}{\Longleftrightarrow} 450 - 10x + 4 = 35x$

$\Leftrightarrow 454 = 45x \Leftrightarrow \boxed{x = \dfrac{454}{45}}$

b) $\sqrt[4]{x} = z \Rightarrow \sqrt{x} = z^2 \Rightarrow z^2 - 2z = 8 \Rightarrow z^2 - 2z - 8 = 0$

$z = 1 \pm \sqrt{1+8} = 1 \pm 3 \begin{cases} 4 \\ -2 \end{cases} \Rightarrow \sqrt[4]{x} = \begin{cases} 4 \\ -2 \end{cases} \Rightarrow$

$\boxed{x = 4^4 = 256}$ Comprovació: $16 - 2 \cdot 4 = 8$

c) $\begin{cases} xy = 6 \\ ax - y + 2 - 3a = 0 \end{cases} \Rightarrow x \cdot (ax + 2 - 3a) = 6$ } ha de tenir solució única

$ax^2 + (2-3a)x - 6 = 0 \rightarrow (2-3a)^2 + 24a = 0 \Rightarrow$

$\Rightarrow 4 + 9a^2 - 12a + 24a = 0 \Rightarrow 9a^2 + 12a + 4 = 0$

$\Rightarrow a = \dfrac{-6 \pm \sqrt{36-36}}{9} = -\dfrac{6}{9} = -\dfrac{2}{3} \Rightarrow$

$\Rightarrow \quad -\dfrac{2}{3}x^2 + 4x - 6 = 0 \Rightarrow -x^2 + 6x - 9 = 0$

$\rightarrow -(x-3)^2 = 0 \Rightarrow \boxed{x = 3 \Rightarrow y = \dfrac{6}{x} = \dfrac{6}{3} = 2}$

3) $p(x) = -x^3 + 6x^2 - 32$

a) Apliquem la regla de Ruffini

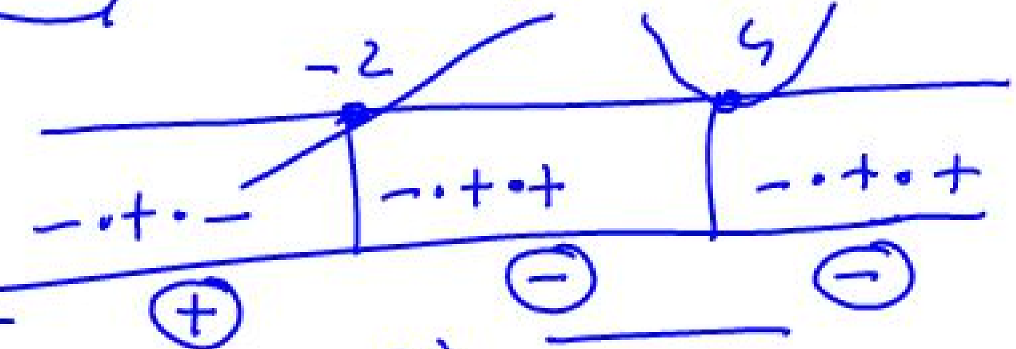
	-1	6	0	-32
4		-4	8	32
	-1	2	8	0

$\Rightarrow p(x) = (x-4)(-x^2+2x+8)$

$-x^2 + 2x + 8 = 0 \Rightarrow x = \dfrac{-1 \pm \sqrt{1+8}}{-1} = \dfrac{-1 \pm 3}{-1} = \begin{cases} -2 \\ 4 \end{cases}$

$\boxed{\text{Arrels: } 4 \text{ i } -2}$ Factors: $p(x) = -(x-4)^2(x+2)$

b) Signe de $p(x)$



$p(x) > 0$ en $(-\infty, -2)$
$p(x) < 0$ en $(-2, 4) \cup (4, +\infty)$
$p(x) = 0$ en $x = -2$ i $x = 4$

c) $-x^3 + 6x^2 - 32 = k$ ha de tenir arrel doble

	-1	6	0	$-32-k$
a		$-a$	$6a-a^2$	$6a^2-a^3$
	-1	$6-a$	$6a-a^2$	$p(a)-k=0$
a		$-a$	$6a-2a^2$	
	-1	$6-2a$	$12a-3a^2=0$	

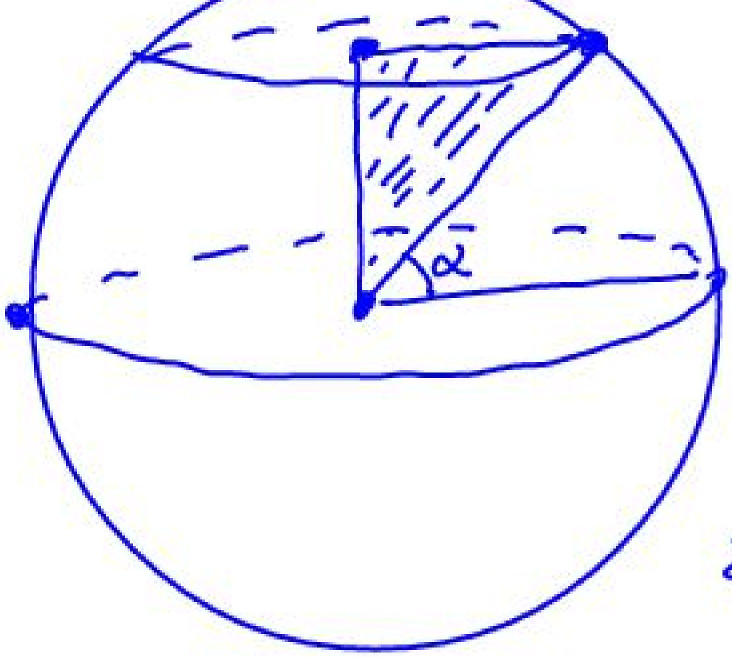
$12a - 3a^2 = 0 \Rightarrow 3a(4-a) = 0 \Rightarrow$

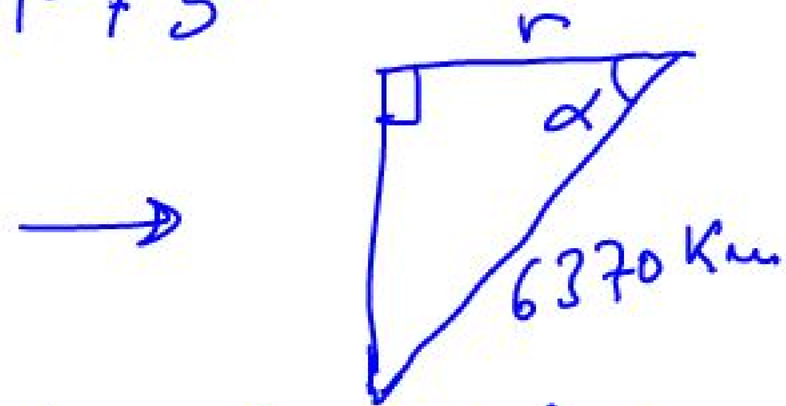
$\Rightarrow a = 0$ o $a = 4$

A partir del signe de $p(x)$ podem dir que hi ha $\Bigg\{$ un mínim en el punt $(0, p(0)) = (0, -32)$
un màxim en el punt $(4, p(4)) = (4, 0)$

4

$\alpha = 41^\circ 7' 5''$

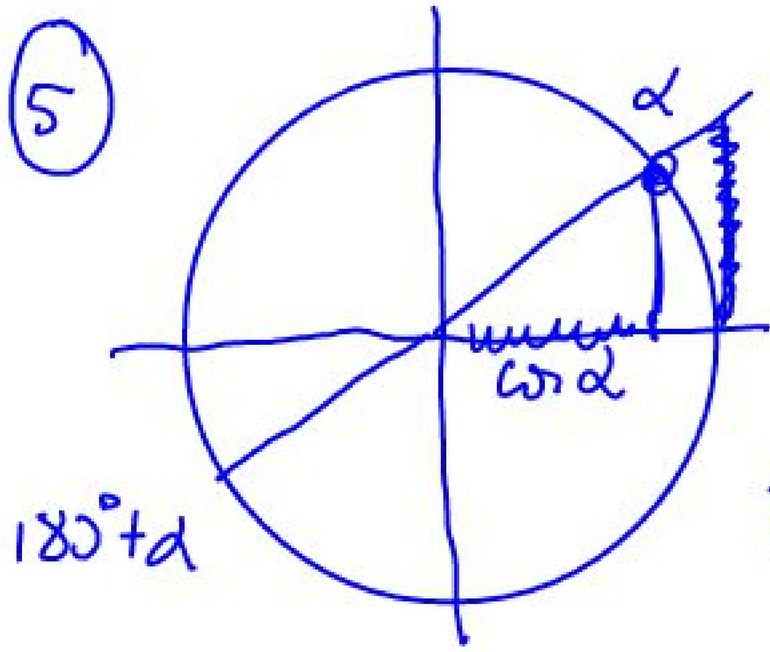




Longitud del paral·lel:

$$2\pi r = 2\pi \cdot 6370 \cdot \cos(41^\circ 7' 5'') \simeq 4.7335 \cdot 6370 \simeq \boxed{30152 \text{ Km}}$$

5



$$\left.\begin{array}{l} \tan(180^\circ+\alpha) = \tan\alpha \\ 1+\tan^2\alpha = \dfrac{1}{\cos^2\alpha} \end{array}\right\} \Rightarrow$$

$$\tan(180^\circ+\alpha) = \sqrt{\frac{1}{\cos^2\alpha} - 1} =$$

$$= \sqrt{\frac{1}{(7/9)^2} - 1} = \sqrt{\frac{81}{49} - 1} = \sqrt{\frac{32}{49}} = \boxed{\frac{4\sqrt{2}}{7}}$$

6 a) $\dfrac{3+i}{2-5i} \cdot \dfrac{2+5i}{2+5i} = \dfrac{(6-5)+(15+2)i}{4+25} = \dfrac{1+7i}{29} = \boxed{\dfrac{1}{29} + \dfrac{17}{29} i}$

b) $(4+2i)^3 = 4^3 + 3\cdot 4^2 \cdot 2i + 3\cdot 4\cdot (2i)^2 + (2i)^3 =$
$= 64 + 96i - 48 - 8i = \boxed{16 + 88i}$

c) $\dfrac{x^3}{(x+3)^2} - \dfrac{9}{x+3} = \boxed{\dfrac{x^3 - 9x - 27}{(x+3)^2}}$

	1	0	−9	−27
−3		−3	9	0
	1	−3	0	−27

no es pot simplificar perquè $x=-3$ no és arrel del numerador