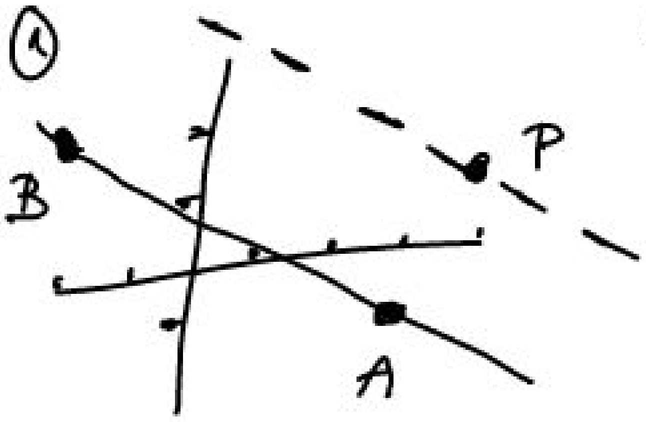
1r BAT II - 070414

①



$A(3,-1)$
$B(-2,2)$
$P(4,1)$

a) $\dfrac{x-3}{3-(-2)} = \dfrac{y+1}{-1-2} \iff \boxed{\dfrac{x-3}{5} = \dfrac{y+1}{-3}}$

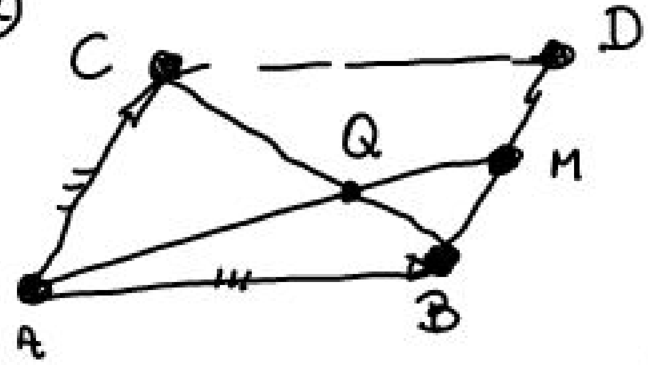
$x=0 \Rightarrow \dfrac{-3}{5} = \dfrac{y+1}{-3} \Rightarrow y = \dfrac{9}{5} - 1 = \dfrac{4}{5}$

$y=0 \Rightarrow \dfrac{x-3}{5} = -\dfrac{1}{3} \Rightarrow x = -\dfrac{5}{3} + 3 = \dfrac{4}{3}$

$\boxed{\dfrac{x}{4/3} + \dfrac{y}{4/5} = 1}$

b) Vector director $= (5,-3) \Rightarrow 3x+5y+k=0$
$P(4,1) \in$ recta $\Rightarrow 3\cdot 4 + 5\cdot 1 + k = 0 \Rightarrow k = -17$ $\Big\} \Rightarrow \boxed{3x+5y-17=0}$

②



a) En la referència $\{A, \vec{AB}, \vec{AC}\}$

$\vec{AA} = 0\cdot\vec{AB} + 0\cdot\vec{AC} \Rightarrow A = (0,0)$
$\vec{AB} = 1\cdot\vec{AB} + 0\cdot\vec{AC} \Rightarrow B = (1,0)$
$\vec{AC} = 0\cdot\vec{AB} + 1\cdot\vec{AC} \Rightarrow C = (0,1)$
$\vec{AD} = 1\cdot\vec{AB} + 1\cdot\vec{AC} \Rightarrow D = (1,1)$
$\vec{AM} = 1\cdot\vec{AB} + \frac{1}{2}\vec{AC} \Rightarrow M = (1, 1/2)$

b) $r_{AM}: \ y = \frac{1}{2}x$
$r_{BC}: \ y = \frac{1}{-1}x + 1 \rightarrow y = -x+1$
$\Big\{ r_{AM} \cap r_{BC}: \frac{1}{2}x = -x+1 \Rightarrow \begin{cases} x = \frac{2}{3} \\ y = \frac{1}{3} \end{cases}$

$\boxed{Q = \left(\frac{2}{3}, \frac{1}{3}\right)}$

$\vec{QC} = (0,1) - \left(\frac{2}{3}, \frac{1}{3}\right) = \left(-\frac{2}{3}, \frac{2}{3}\right)$
$\vec{BC} = (0,1) - (1,0) = (-1,0)$ $\Big\} \Rightarrow \boxed{\vec{QC} = \frac{2}{3}\vec{BC}}$

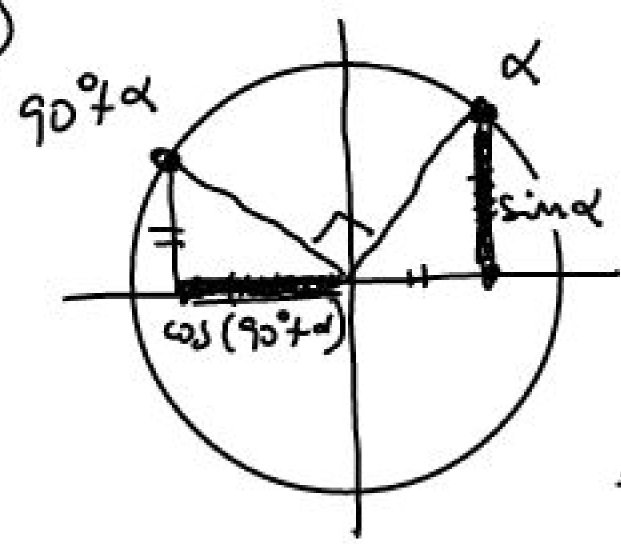
Resolució 2: ΔAQC i ΔMQB són semblants amb raó de semblança $AC/BM = 2$. Per tant, $\dfrac{QC}{BQ} = 2$.

Llavors, $\vec{BC} = \vec{QC} + \vec{BQ} = \vec{QC} + \frac{1}{2}\vec{QC} \Rightarrow \boxed{\vec{QC} = \frac{2}{3}\vec{BC}}$
$\vec{AQ} = \vec{AB} + \frac{1}{2}\vec{QC} = (1,0) + \frac{1}{2}\cdot\frac{2}{3}\vec{BC} = (1,0) + \frac{1}{3}(-1,1) = \boxed{\left(\frac{2}{3}, \frac{1}{3}\right) = Q}$

③ $1 + \cos(2x) = \cos x \Rightarrow 1 + \cos^2 x - (1 - \cos^2 x) = \cos x \Rightarrow 2\cos^2 x - \cos x = 0$

$\Rightarrow \cos x\,(2\cos x - 1) = 0 \Rightarrow \begin{cases} \cos x = 0 \\ \text{o} \\ \cos x = \frac{1}{2} \end{cases} \Rightarrow \boxed{\begin{array}{l} x = \frac{\pi}{2} + k\pi \\ \text{o} \\ x = \pm\frac{\pi}{3} + 2k\pi \end{array}}$

④



$\cos\alpha = \frac{3}{5} \Rightarrow \cos(90^\circ+\alpha) = -\sin\alpha =$

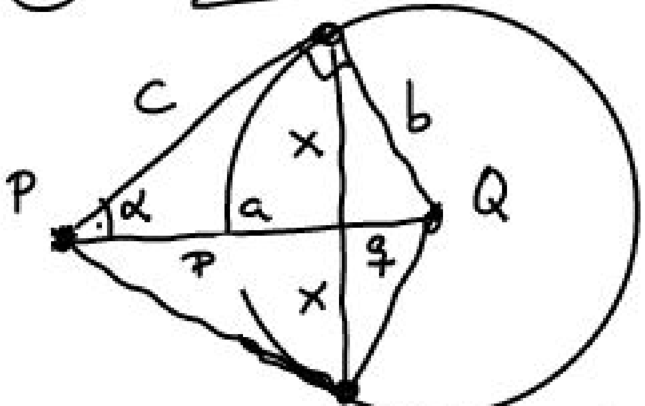
$= -\sqrt{1-\left(\frac{3}{5}\right)^2} = -\sqrt{\frac{16}{25}} = \boxed{-\frac{4}{5}}$

Calculadora:

$\cos\alpha = \frac{3}{5} \Rightarrow \alpha = \arccos(3/5) = 53^\circ\, 7'\, 48.37''$

$\Rightarrow \cos(90^\circ+\alpha) = \cos 143^\circ\, 7'\, 48.37'' = -0.8$

⑤ Resolució 1:



Pels teoremes del catet i de l'altura

$x^2 = p\cdot q$
$c^2 = p\cdot a$
$b^2 = q\cdot a$
$\Rightarrow p\cdot q\cdot a^2 = b^2c^2$

$\Rightarrow x^2\cdot a^2 = b^2c^2 \Rightarrow \boxed{2x = \frac{2bc}{a}}$

Resolució 2:

$\sin\alpha = \frac{x}{c}$
$\sin\alpha = \frac{b}{a}$
$\Rightarrow \frac{x}{c} = \frac{b}{a}$

$\Rightarrow \boxed{2x = \frac{2bc}{a}}$

Resolució 3:

$4x^2 = c^2+c^2-2c^2\cos(2\alpha) = 4x^2 = 2c^2\left(1-(\cos^2\alpha-\sin^2\alpha)\right)$

$= 2c^2\cdot 2\sin^2\alpha = 4c^2\cdot\left(\frac{b}{a}\right)^2 \Rightarrow \boxed{2x = \frac{2cb}{a}}$

⑥ $\sqrt[3]{i} = \sqrt[3]{1_{90^\circ}} =$

$\rightarrow 1_{30^\circ} = \cos 30^\circ + i\sin 30^\circ = \frac{\sqrt{3}}{2}+\frac{1}{2}i$

$\rightarrow 1_{150^\circ} = \cos 150^\circ + i\sin 150^\circ = -\frac{\sqrt{3}}{2}+\frac{1}{2}i$

$\rightarrow 1_{270^\circ} = \cos 270^\circ + i\sin 270^\circ = -i$

⑦ a) $z = x+yi \Rightarrow z^2+\bar{z}^2 = (x+yi)^2+(x-yi)^2 = 2(x^2-y^2)+2xyi-2xyi = \boxed{x^2-y^2}$

b) $z^2+\bar{z}^2=0 \Rightarrow x^2-y^2=0 \Rightarrow (x+y)(x-y)=0 \Rightarrow$ $x+y=0$ o $x-y=0$