

$$1) a) \frac{\sqrt[3]{a^3 ab}}{\sqrt[3]{a b^2}} = \sqrt[6]{\frac{a^3 \cdot ab}{a^2 b^4}} = \sqrt[6]{\frac{a^2}{b^3}} = \frac{\sqrt[3]{a^2}}{\sqrt[3]{b^3}} = \frac{\sqrt[3]{a^2}}{\sqrt[3]{b^3}} = \frac{\sqrt[3]{a^2 b^3}}{b}$$

$$b) \frac{\sqrt{84} + \sqrt{75} - \sqrt{729}}{2\sqrt{3}} = \frac{2\sqrt{21} + 5\sqrt{3} - 3\sqrt{3}}{2\sqrt{3}} = \frac{(2\sqrt{7} + 5 - 3)\sqrt{3}}{2\sqrt{3}} = \frac{\sqrt{12}}{2} = \frac{2\sqrt{7} + 2}{2} = \sqrt{7} + 1$$

$$2) a) 4x^4 + 7x^2 - 2 = 0 \Rightarrow x^2 = \frac{-7 \pm \sqrt{49 + 32}}{8} = \frac{-7 \pm 9}{8} \Rightarrow \begin{matrix} \frac{1}{4} \\ -2 \end{matrix}$$

$$x = \pm \sqrt{\frac{1}{4}} = \pm \frac{1}{2}, \quad x = \pm \sqrt{-2} \notin \mathbb{R}$$

$$b) \frac{x - \sqrt{1-3x}}{3} = x + 2 \Leftrightarrow x - \sqrt{1-3x} = 3x + 6 \Leftrightarrow -\sqrt{1-3x} = 2x + 6$$

$$\Rightarrow 1 - 3x = 4x^2 + 24x + 36 \Leftrightarrow 4x^2 + 27x + 35 = 0$$

$$x = \frac{-27 \pm \sqrt{729 - 560}}{8} = \frac{-27 \pm 13}{8} \Rightarrow \begin{matrix} -\frac{14}{8} = -\frac{7}{4} \\ -\frac{40}{8} = -5 \end{matrix}$$

Comprovació:

$$\left\{ \begin{array}{l} \frac{-7/4 - \sqrt{25/4}}{3} = \frac{-7/4 - 5/2}{3} = -\frac{17}{12} \\ -7/4 + 2 = 1/2 \end{array} \right\} \Rightarrow x = -7/4 \text{ no s'ha de considerar}$$

$$\left\{ \begin{array}{l} \frac{-5 - \sqrt{16}}{3} = -3 \\ -5 + 2 = -3 \end{array} \right\} \Rightarrow \boxed{x = -5}$$

$$c) \left\{ \begin{array}{l} 2 - x > 3 \\ 3x + 8 < 1 \end{array} \right\} \Leftrightarrow \left\{ \begin{array}{l} 2 - 3 > x \\ 3x < 1 - 8 \end{array} \right\} \Leftrightarrow \left\{ \begin{array}{l} x < -1 \\ x < -7/3 \end{array} \right\} \Leftrightarrow \boxed{x < -7/3}$$

Solució:  $(-\infty, -7/3)$

$$3) \begin{cases} xy = 4 \\ 2x + y = k \end{cases} \Rightarrow x(k - 2x) = 4 \Rightarrow -2x^2 + kx - 4 = 0 \Rightarrow$$

$$\Rightarrow x = \frac{-k \pm \sqrt{k^2 - 32}}{-4}$$

$$a) \text{ Solució única } \Leftrightarrow k^2 - 32 = 0 \Leftrightarrow \boxed{k = \pm \sqrt{32}} = \pm 4\sqrt{2}$$

$$b) \text{ Cap solució } \Leftrightarrow k^2 - 32 < 0 \Leftrightarrow \boxed{-4\sqrt{2} < k < 4\sqrt{2}}$$

④  $p(x) = 6x^3 + 13x^2 + x - 2$

a) Apliquem la regla de Ruffini

$$-2 \begin{array}{r|rrrr} & 6 & 13 & 1 & -2 \\ & & -12 & -2 & 2 \\ \hline & 6 & 1 & -1 & 0 \end{array}$$

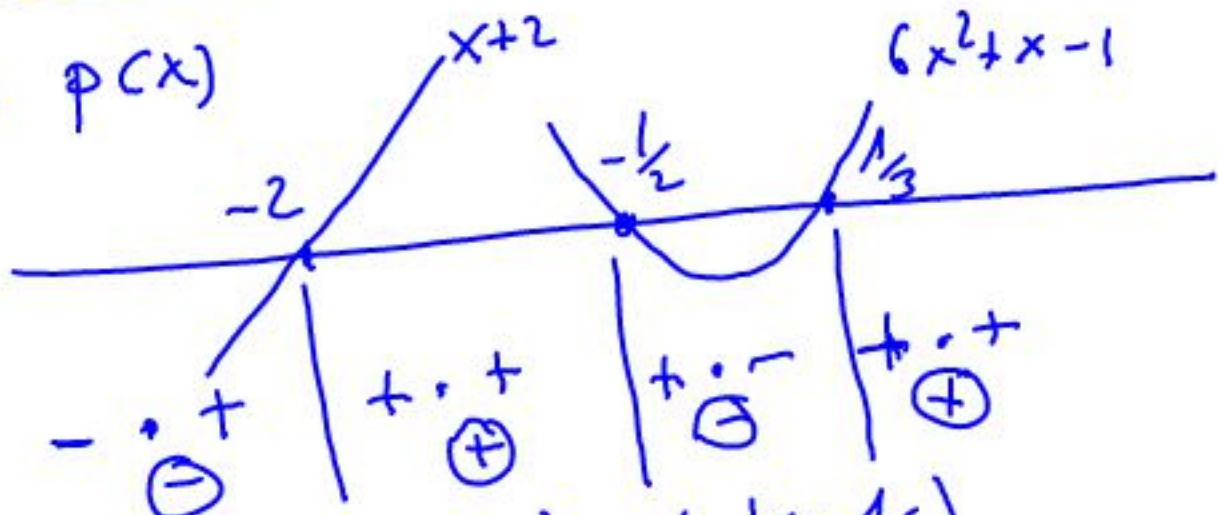
$p(x) = (x+2)(6x^2+x-1) = 0$

$\Rightarrow 6x^2+x-1=0 \Rightarrow x = \frac{-1 \pm \sqrt{1+24}}{12} = \begin{matrix} \rightarrow 1/3 \\ \rightarrow -1/2 \end{matrix}$   
 $x+2=0 \Rightarrow x=-2$

Arrels:  $x = -2, x = 1/3, x = -1/2$

Descomposició factorial:  $(x+2) \cdot 6 \cdot (x-1/3)(x+1/2) = (x+2)(3x-1)(2x+1)$

b) Signe de  $p(x)$



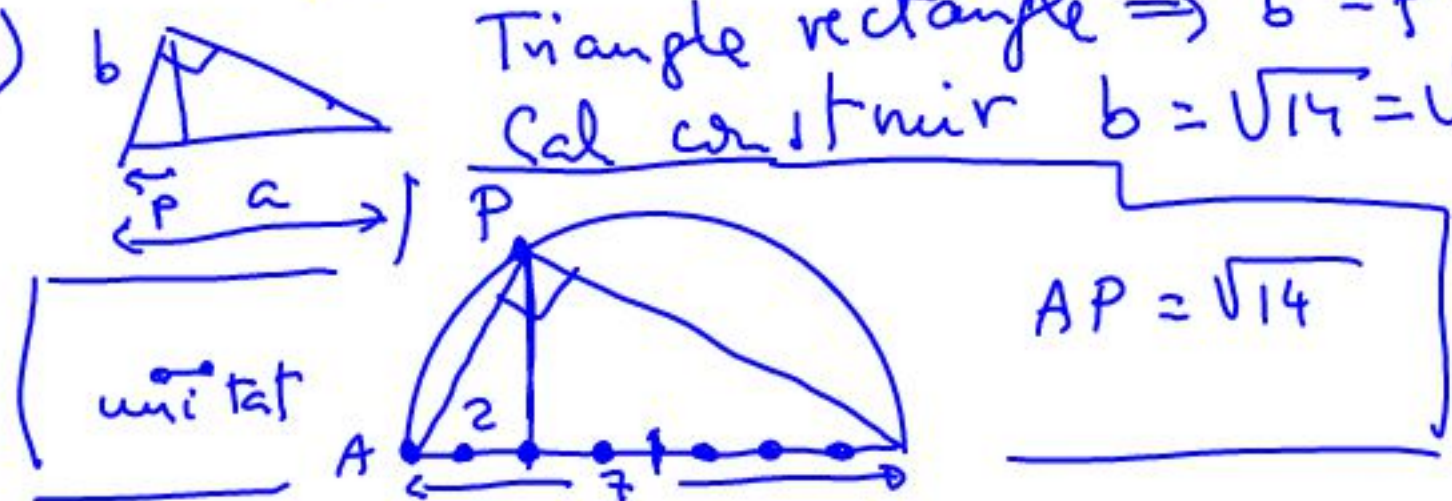
$p(x) < 0$  en  $(-\infty, -2) \cup (-1/2, 1/3)$   
 $p(x) > 0$  en  $(-2, -1/2) \cup (1/3, +\infty)$

⑤ b)  $\sum_{k=0}^{1000} \binom{1000}{k} 1^{1000-k} \cdot 4^k = (1+4)^{1000} = \boxed{5^{1000}}$

a) 
$$a \begin{array}{r|rrrrrr} & 1 & 0 & 0 & \dots & 0 & 0 & -a^m \\ & & a & a^2 & \dots & a^{m-2} & a^{m-1} & a^m \\ \hline & 1 & a & a^2 & \dots & a^{m-2} & a^{m-1} & 0 \end{array}$$
  
 $p(x) = x^{m-1} + ax^{m-2} + a^2x^{m-3} + \dots + a^{m-2}x + a^{m-1} = \sum_{k=1}^m a^{k-1} x^{m-k}$

c)  $(\sqrt{6-\sqrt{11}} + \sqrt{6+\sqrt{11}})^2 = 6-\sqrt{11} + 6+\sqrt{11} + 2\sqrt{36-11} = 12+10 = 22 \Rightarrow$   
 $\Rightarrow \sqrt{6-\sqrt{11}} + \sqrt{6+\sqrt{11}} = \sqrt{22}$

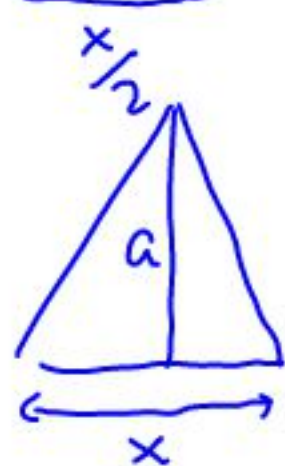
d) Triangle rectangle  $\Rightarrow b^2 = p \cdot a \Rightarrow b = \sqrt{p \cdot a}$   
 Cal construir  $b = \sqrt{14} = \sqrt{2 \cdot 7}$



6

$$S = 36 \text{ m}^2$$

$$V = 8 \text{ m}^3$$



$$36 = x^2 + \frac{4 \cdot x \cdot a}{2}$$

$$8 = \frac{x^2 \cdot h}{3}$$

$$h = \sqrt{a^2 - \frac{x^2}{4}} = \frac{\sqrt{4a^2 - x^2}}{2}$$

$$36 = x^2 + 2ax$$

$$24 = \frac{x^2 \sqrt{4a^2 - x^2}}{2}$$

$$2a = \frac{36 - x^2}{x}$$

$$48 = x^2 \sqrt{4a^2 - x^2}$$

$$48 = x^2 \sqrt{\frac{(36 - x^2)^2}{x^2} - x^2} \Rightarrow 48 = x \sqrt{1296 - 72x^2 + x^4 - x^4}$$

$$2304 = x^2 (1296 - 72x^2) \Rightarrow 128 = x^2 (72 - 4x^2)$$

$$\Rightarrow x^4 - 18x^2 + 32 = 0 \Rightarrow x^2 = 9 \pm \sqrt{81 - 32}$$

$$\Rightarrow x^2 = 9 \pm 7 = \begin{cases} 16 \\ 2 \end{cases} \Rightarrow x = 4, x = \sqrt{2}$$

$$x = 4 \text{ m} \Rightarrow a = \frac{36 - 16}{8} = \frac{20}{8} = \frac{5}{2} = 2,5 \text{ m}$$

$$x = \sqrt{2} \text{ m} \Rightarrow a = \frac{36 - 2}{2\sqrt{2}} = \frac{34}{2\sqrt{2}} = \frac{17\sqrt{2}}{2} \approx 12,021 \text{ m}$$

$\hookrightarrow \approx 1,414 \text{ m}$

• Comprovació:

$$x = 4 \text{ m} \Rightarrow \left\{ \begin{array}{l} S = 16 + 2 \cdot 2,5 \cdot 4 = 16 + 20 = 36 \text{ m}^2 \\ V = \frac{16 \cdot \sqrt{6,25 - 4}}{3} = \frac{16 \cdot 1,5}{3} = 8 \text{ m}^3 \end{array} \right.$$

$$x = \sqrt{2} \text{ m} \Rightarrow \left\{ \begin{array}{l} S = 2 + 2 \cdot \sqrt{2} \cdot \frac{17\sqrt{2}}{2} = 2 + 34 = 36 \text{ m}^2 \\ V = \frac{2 \cdot \sqrt{17^2/2 - 1/2}}{3} = \frac{2 \cdot \sqrt{144}}{3} = 8 \text{ m}^3 \end{array} \right.$$