(1)

$30-10x^2$

	$-\sqrt{3}$		$\sqrt{3}$	
signe(f')	$-$		$+$		$-$
monotonia(f)	↘		↗		↘

$f'(-\sqrt{3})=0$, $f'(\sqrt{3})=0$

Mínim local en $x=-\sqrt{3}$ i $f(-\sqrt{3})=-\frac{5\sqrt{3}}{3}$

Màxim " en $x=\sqrt{3}$ i $f(\sqrt{3})=\frac{5\sqrt{3}}{3}$

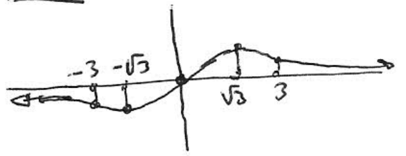
$20x$, x^2-9

	-3		0		3	
signe(f'')	$-$		$+$		$-$		$+$
concavitat(f)	$\cap$		$\cup$		$\cap$		$\cup$

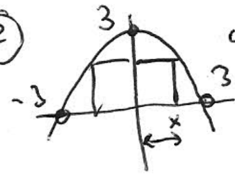
$f''(-3)=0$, $f''(0)=0$, $f''(3)=0$ $\Big\}$ Punts d'inflexió $(-3,-\frac{5}{2})$, $(0,0)$, $(3,\frac{5}{2})$

A.H. $\lim_{x\to\pm\infty} f(x)=\lim \frac{10/x}{1+3/x^2}=\frac{0}{1+0}=0 \Leftrightarrow \boxed{y=0}$

Talls eixos: $(0,0)$



(2)



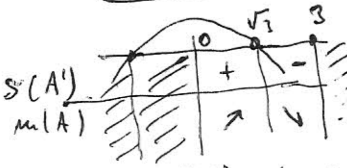
a) Paràbola $\to p(x)=a(x+3)(x-3)$

$p(x)=a(x^2-9)$

$3=p(0)=a\cdot(-9) \Rightarrow a=-\frac{1}{3}$

$p(x)=3-\frac{x^2}{3} \Rightarrow A(x)=2x\left(3-\frac{x^2}{3}\right)=6x-\frac{2x^3}{3}, \quad 0<x<3$

b) $A'(x)=6-2x^2=0 \Rightarrow x=\pm\sqrt{3}$



$\Rightarrow$ Màxim absolut: $(\sqrt{3}, A(\sqrt{3}))=(\sqrt{3}, 4\sqrt{3})$

L'altura de la porta per a l'àrea màxima $A(\sqrt{3})$ és $3-\frac{\sqrt{3}^2}{3}=3-1=\boxed{2}$

c) $2\int_0^3\left(3-\frac{x^2}{3}\right)dx-4\sqrt{3}=2\left[3x-\frac{x^3}{9}\right]_0^3-4\sqrt{3}=2(9-3)-4\sqrt{3}=\boxed{12-4\sqrt{3}}\simeq 5{,}072$

(3) $F(x)=\int_0^x t\cdot e^t dt \Rightarrow F'(x)=xe^x, \quad 0<x<+\infty \Rightarrow F'(x)=xe^x>0 \Rightarrow$ F creixent

(4) $s(x)=2x+1$ divisor de $p(x) \Rightarrow 0=p(-\frac{1}{2})=-\frac{1}{2}+\frac{a}{4}-\frac{b}{2}-15 \Rightarrow a-2b=62$ $\Big\} \Rightarrow a=-12, \; b=-37$

$x=1$ punt d'inflexió $\Rightarrow p''(1)=0$ $\Big\} \Rightarrow 24+2a=0 \Rightarrow a=-12$

$p'(x)=12x^2+2ax+b$

$p''(x)=24x+2a$

Llavors, $p(x)=4x^3-12x^2-37x-15$ i una de les seves arrels és $x=-\frac{1}{2}$

Altres arrels:

	4	−12	−37	−15
$-\frac{1}{2}$		−2	7	15
	4	−14	−30	0

$\to 2x^2-7x-15=0 \Rightarrow x=\frac{7\pm\sqrt{49+120}}{4}=5, \; -\frac{3}{2}$

(*) Darrere

(5) $f(x)=x\cdot\ln x \Rightarrow f'(x)=\ln x+x\cdot\frac{1}{x}=\ln x+1 \Rightarrow f''(x)=\frac{1}{x}$ (per a $n=2$ cert)

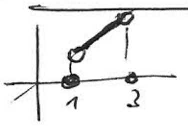
$f^{(n)}(x)$ cert $\overset{?}{\Rightarrow} f^{(n+1)}(x)$ cert: $f^{(n+1)}(x)=\left[\frac{(-1)^n(n-2)!}{x^{n-1}}\right]'=(-1)^n(n-2)!\,(-n+1)\cdot x^{-n+1-1}$

$=(-1)^n(-1)(n-1)(n-2)!\,x^{-n}=\frac{(-1)^{n+1}(n-1)!}{x^n}$

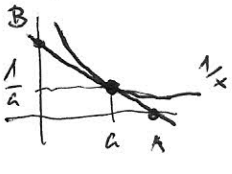
(6) $\lim_{x\to0^+} xe^{1/x}=0\cdot e^{+\infty}=0\cdot(+\infty)=$ (Indeterminat) $=\lim_{x\to0^+}\frac{e^{1/x}}{1/x}\underset{\text{l'Hôpital}}{=}\lim_{x\to0^+}\frac{e^{1/x}\cdot(-1/x^2)}{-1/x^2}=\lim_{x\to0^+}e^{1/x}=e^{+\infty}=+\infty$

$\lim_{x\to0^-} xe^{1/x}=0\cdot e^{-\infty}=0\cdot\frac{1}{e^{+\infty}}=0\cdot\frac{1}{+\infty}=0\cdot0=\boxed{0}$ $\longrightarrow$ Discontinuïtat asimptòtica

(7) $f(x)=x$ en l'interval que es vulgui, per exemple $(1,3)$



(8)



$y-\frac{1}{a}=-\frac{1}{a^2}(x-a)$ $\Big\{$ A: $y=0 \Rightarrow a=x-a \Rightarrow x=2a$; B: $x=0 \Rightarrow y=\frac{1}{a}+\frac{1}{a}=\frac{2}{a}$ $\Big\}$ Àrea $=\frac{1}{2}2a\cdot\frac{2}{a}=\boxed{2}$

9) $f(x)=\dfrac{x}{\sqrt{1+x}}$; $\sqrt{1+x}=t \Rightarrow x=t^2-1 \Rightarrow x'(t)=2t$

$$\int \frac{t^2-1}{t}\,2t\,dt = 2\int (t^2-1)\,dt = 2\frac{t^3}{3}-2t+K \Rightarrow$$

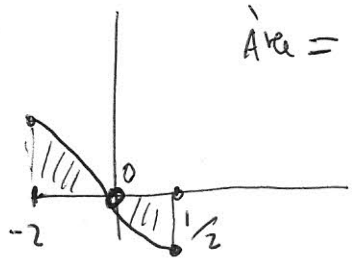
$$\Rightarrow \int \frac{x}{\sqrt{1+x}}\,dx = 2\left[\frac{(\sqrt{1+x})^3}{3}-\sqrt{1+x}\right]+K = \frac{2\sqrt{1+x}}{3}(1+x-3)+K = \boxed{\frac{2}{3}(x-2)\sqrt{1+x}+K}$$

$3=\frac{2}{3}\cdot 6\cdot 3+K \Rightarrow$

$\Rightarrow K=-9$

$$\boxed{F(x)=\frac{2}{3}(x-2)\sqrt{1+x}-9}$$

10) $$\text{Àrea} = \int_{-2}^{0}\frac{x}{(x-1)^3}\,dx - \int_{0}^{1/2}\frac{x}{(x-1)^3}\,dx$$



$$\int \frac{x}{(x-1)^3}\,dx = \int\left(\frac{A}{x-1}+\frac{B}{(x-1)^2}+\frac{C}{(x-1)^3}\right)dx$$

$$x = A(x-1)^2+B(x-1)+C$$

$x=1 \Rightarrow C=1$
$x=2 \Rightarrow A+B+C=2$
$x=0 \Rightarrow A-B+C=0$
$\Rightarrow$ $C=1$, $A+B=1$, $A-B=-1$ $\Rightarrow$ $C=1$, $A=0$, $B=1$

$$\int \frac{x}{(x-1)^3}\,dx = \int \frac{1}{(x-1)^2}\,dx + \int \frac{1}{(x-1)^3}\,dx = -\frac{1}{(x-1)}-\frac{1}{2(x-1)^2}+K$$

$$\text{Àrea} = \left[-\frac{1}{x-1}-\frac{1}{2(x-1)^2}\right]_{-2}^{0} - \left[-\frac{1}{x-1}-\frac{1}{2(x-1)^2}\right]_{0}^{1/2} = \left[\left(1-\frac{1}{2}\right)-\left(\frac{1}{3}-\frac{1}{18}\right)\right]-\left[(2-2)-\left(1-\frac{1}{2}\right)\right]$$

$$=\left(\frac{1}{2}-\frac{5}{18}\right)-\left(0-\frac{1}{2}\right)=\frac{4}{18}+\frac{1}{2}=\boxed{\frac{13}{18}\simeq 0{,}72}$$

(*) Si es contestava que no era cert, de manera justificada, era correcta.

La que era certa: $f^{(n)}(x)=\dfrac{(-1)^n\,(n-2)!}{x^{n-1}}$