2BAT - 11/5/16

1

$$\left(\begin{array}{ccc|c} 2 & a & -4 & 5 \\ 1 & -1 & -5 & 3 \\ a & 7 & -a & 7 \end{array}\right)$$

$$\det A = 2a - 5a^2 - 28 - 4a + 70 + a^2$$
$$= -4a^2 - 2a + 42 = 0$$
$$\det A = 0 \iff a = \frac{1 \pm \sqrt{169}}{-4} = \begin{cases} -\frac{7}{2} \\ 3 \end{cases}$$

$\boxed{a=3}$ $M = \begin{vmatrix} 2 & 3 \\ 1 & -1 \end{vmatrix} \neq 0$ i $\det A = 0 \Rightarrow r(A) = 2$

$M \neq 0$ i $\det A = 0$ i $\begin{vmatrix} 2 & 3 & 5 \\ 1 & -1 & 3 \\ 3 & 7 & 7 \end{vmatrix} = 0 \Rightarrow r(A|B) = 2$

$\Rightarrow$ Sistema compatible indeterminat

$\boxed{a=-\frac{7}{2}}$ $M = \begin{vmatrix} 2 & -7/2 \\ 1 & -1 \end{vmatrix} \neq 0$ i $\det A = 0 \Rightarrow r(A) = 2$

$M \neq 0$ i $\det A = 0$ i $\begin{vmatrix} 2 & -7/2 & 5 \\ 1 & -1 & 3 \\ -7/2 & 7 & 7 \end{vmatrix} \neq 0 \Rightarrow r(A|B) = 3$

$\Rightarrow$ Sistema incompatible

$\boxed{a \neq 3 \text{ i } a \neq -\frac{7}{2}}$ $\det A \neq 0 \Rightarrow r(A) = 3 = r(A|B) \Rightarrow$ Sistema compatible determ.

Resolució per $a=3$: $\boxed{z = \lambda}$

$$\left(\begin{array}{cc|c} 2 & 3 & 5+4\lambda \\ 1 & -1 & 3+5\lambda \end{array}\right) \Rightarrow$$

$$x = \frac{\begin{vmatrix} 5+4\lambda & 3 \\ 3+5\lambda & -1 \end{vmatrix}}{-2-3} = \frac{-14-19\lambda}{-5} = \boxed{\frac{14}{5} + \frac{19}{5}\lambda}$$

$$y = \frac{\begin{vmatrix} 2 & 5+4\lambda \\ 1 & 3+5\lambda \end{vmatrix}}{-2-3} = \frac{1+6\lambda}{-5} = \boxed{-\frac{1}{5} - \frac{6}{5}\lambda}$$

3A a) En general $A \cdot B$ i $B \cdot A$ poden ser iguals o diferents. Per tant,

$(A+B)^2 = (A+B)(A+B) = A^2 + A\cdot B + B\cdot A + B^2$ no és idèntic a $A^2 + AB + AB + B^2 = A^2 + 2AB + B^2$

b)
$$A^2 = \begin{pmatrix} 1 & 1 \\ 2 & 3 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 2 & 3 \end{pmatrix} = \begin{pmatrix} 3 & 4 \\ 8 & 11 \end{pmatrix}$$
$$2A\cdot B = 2\begin{pmatrix} 1 & 1 \\ 2 & 3 \end{pmatrix}\begin{pmatrix} 4 & 1 \\ 2 & 1 \end{pmatrix} = \begin{pmatrix} 12 & 4 \\ 28 & 10 \end{pmatrix}$$
$$B^2 = \begin{pmatrix} 4 & 1 \\ 2 & 1 \end{pmatrix}\begin{pmatrix} 4 & 1 \\ 2 & 1 \end{pmatrix} = \begin{pmatrix} 18 & 5 \\ 10 & 3 \end{pmatrix}$$
$$\boxed{A^2 + 2AB + B^2 = \begin{pmatrix} 33 & 13 \\ 46 & 24 \end{pmatrix}}$$

3B

$$A = \begin{pmatrix} 3 & 1 \\ -1 & 1 \end{pmatrix},\ B\begin{pmatrix} 4 & 3 \\ -1 & 0 \end{pmatrix},\ C = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$$

$$AX + B = C \iff AX = C - B \iff A^{-1}AX = A^{-1}(C-B) \iff X = A^{-1}(C-B)$$

$$A^{-1}: \left(\begin{array}{cc|cc} 3 & 1 & 1 & 0 \\ -1 & 1 & 0 & 1 \end{array}\right) \xrightarrow{3F_2+F_1 \to F_2} \left(\begin{array}{cc|cc} 3 & 1 & 1 & 0 \\ 0 & 4 & +1 & 3 \end{array}\right) \xrightarrow{4F_1 - F_2 \to F_1} \left(\begin{array}{cc|cc} 12 & 0 & 3 & -3 \\ 0 & 4 & +1 & 3 \end{array}\right) \xrightarrow[F_2/4 \to F_2]{F_1/12 \to F_1} \left(\begin{array}{cc|cc} 1 & 0 & 3/12 & -1/4 \\ 0 & 1 & +1/4 & 3/4 \end{array}\right)$$

$$\underbrace{\begin{pmatrix} 3/12 & -1/4 \\ +1/4 & 3/4 \end{pmatrix}}_{A^{-1}}$$

$$X = \begin{pmatrix} 1/4 & -1/4 \\ +1/4 & 3/4 \end{pmatrix}\begin{pmatrix} -3 & -1 \\ 3 & 1 \end{pmatrix} = \begin{pmatrix} -\frac{3}{4} - \frac{3}{4} & -\frac{1}{4} - \frac{1}{4} \\ -\frac{3}{4} + \frac{9}{4} & -\frac{1}{4} + \frac{3}{4} \end{pmatrix} = \boxed{\begin{pmatrix} -\frac{3}{2} & -\frac{1}{2} \\ +\frac{3}{2} & \frac{1}{2} \end{pmatrix}}$$

② a) Són coplanàries, si es tallen o són paral·leles.

- Mirem si es tallen:

$s \cap r$: Substituïm $(x,y,z) \in r$ en s.

$$\begin{cases} -1-\lambda-2(1+3\lambda)-(-2-\lambda)+1=0 \iff -6\lambda=0 \iff \lambda=0 \\ -1-\lambda+6(1+3\lambda)+(-2-\lambda)-3=0 \iff 16\lambda=0 \iff \lambda=0 \end{cases}$$

Per tant es tallen en

$$\left.\begin{aligned} x &= -1-0=-1 \\ y &= 1+3\cdot 0 = 1 \\ z &= -2-0=-2 \end{aligned}\right\} \Rightarrow \text{punt } A(-1,1,-2)$$

i això implica que són coplanàries.

b) Cercarem els ~~[illegible]~~ vectors directors i l'angle que formen serà l'angle de les rectes.

$$\vec{u}_r = (-1,3,-1)$$

$$\vec{u}_s = \begin{vmatrix} \vec{e}_1 & \vec{e}_2 & \vec{e}_3 \\ 1 & -2 & -1 \\ 1 & 6 & 1 \end{vmatrix} = (4,-2,8) \text{ o, més curt, } (2,-1,4)$$

Llavors l'angle α de les dues rectes serà el que satisfà:

$$\cos\alpha = \frac{|\vec{u}_r \cdot \vec{u}_s|}{|\vec{u}_r|\cdot|\vec{u}_s|} = \frac{|(2,-1,4)\cdot(-1,3,1)|}{\sqrt{2^2+(-1)^2+4^2}\,\sqrt{(-1)^2+3^2+(-1)^2}} = \frac{|-2-3-4|}{\sqrt{21}\cdot\sqrt{11}} = \frac{9}{\sqrt{231}}$$

Finalment:

$$\boxed{\alpha = \arccos\left(\frac{9}{\sqrt{231}}\right) \simeq 53^\circ\, 41'\, 23.31''}$$

a) Equació del pla que conté les dues rectes

$$\left.\begin{aligned} &\det(\vec{u}_r, \vec{u}_s, \vec{AX})=0 \\ &X(x,y,z) \in \text{pla} \end{aligned}\right\} \quad \begin{vmatrix} -1 & 2 & x+1 \\ 3 & -1 & y-1 \\ -1 & 4 & z+2 \end{vmatrix} = 0 \iff 11(x+1)+2(y-1)-5(z+2)=0$$

$$\iff \boxed{11x+2y-5z-1=0}$$