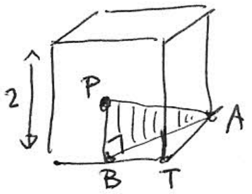
4eso A // 15/2/16

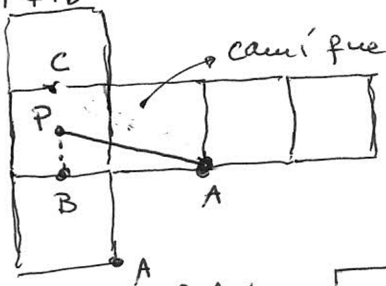
①



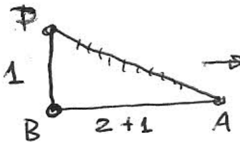
a) $\left.\begin{array}{l} AP = \sqrt{AB^2+BP^2} \\ AB^2 = 2^2+1^2 = 5 \end{array}\right\} \Rightarrow AP = \sqrt{5+1^2} = \boxed{\sqrt{6}\ m \simeq 2{,}449\, m}$

AT^2+TB^2

b) i c)



$\rightarrow$



$\rightarrow \left\{ \begin{array}{l} AC = \sqrt{3^2+1^2} \\ = \sqrt{9+1} \\ \boxed{= \sqrt{10}\, m \\ \simeq 3{,}162\, m} \end{array}\right.$

d) Volum $= 2^3\, m^3 = 8\, m^3 = 8 \cdot 10^3$ litres $= \boxed{8000 \text{ litres}}$

e) Superfície del camp de bàsquet: $28\, m \cdot 15\, m = 420\, m^2$
Volum d'aigua caigut sobre el camp: $420\, m^2 \cdot 14\, l/m^2 = 5880$ litres
N'hi hauria prou amb un contenidor. Concretament l'aigua caiguda ompliria $\frac{5880}{8000} = 0{,}735 = 73{,}5\%$ del contenidor

② a) $4x^2+5x-6=0 \Rightarrow x = \frac{-5 \pm \sqrt{25+96}}{8} = \frac{-5 \pm 11}{8} = \boxed{\begin{array}{l} \frac{6}{8} = \frac{3}{4} \\ -\frac{16}{8} = -2 \end{array}}$

b) $(x-4)(x+4) = x+14 \Rightarrow x^2-16 = x+14 \Rightarrow x^2-x-30=0 \Rightarrow x = \frac{1 \pm \sqrt{1+120}}{2} = \frac{1 \pm 11}{2} = \boxed{\begin{array}{l} 6 \\ -5 \end{array}}$

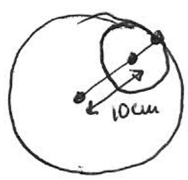
c) $\frac{1}{x} + \frac{12}{x+3} = 5 \Rightarrow x+3+12x = 5x(x+3) \Rightarrow 13x+3 = 5x^2+15x$
$\Rightarrow 5x^2+2x-3=0 \Rightarrow x = \frac{-2 \pm \sqrt{4+60}}{10} = \frac{-2 \pm 8}{10} = \boxed{\begin{array}{l} \frac{6}{10} = \frac{3}{5} \\ -1 \end{array}}$

d) $\left.\begin{array}{l} x+3y=7 \\ 3x+y=3 \end{array}\right\} \Rightarrow \left\{\begin{array}{l} 3x+9y=21 \\ 3x+y=3 \end{array}\right. \Rightarrow \left\{\begin{array}{l} 3x+9y=21 \\ 8y=18 \end{array}\right. \Rightarrow \left\{\begin{array}{l} y = \frac{18}{8} = \frac{9}{4} \\ 3x+9 \cdot \frac{9}{4} = 21 \end{array}\right.$

$\Rightarrow \left\{\begin{array}{l} y = \frac{9}{4} \\ 3x = 21 - \frac{81}{4} = \frac{3}{4} \end{array}\right. \Rightarrow \left\{\boxed{\begin{array}{l} y = \frac{9}{4} \\ x = \frac{3}{4 \cdot 3} = \frac{1}{4} \end{array}}\right.$

e) $\frac{3x-2}{6} - \frac{x}{8} = \frac{1}{4} \xRightarrow{(\cdot 24)} 12x-8-3x=6 \Rightarrow 9x=14 \Rightarrow \boxed{x = \frac{14}{9}}$

③ a)



b) $\left.\begin{array}{l} R = \text{radi de la circumferència gran} \\ r = \text{" " petita} \end{array}\right\} \Rightarrow R - r = 10\, cm$

$\left.\begin{array}{l} \text{Àrea de la circumferència gran: } \pi R^2 \\ \text{" " petita: } \pi r^2 \end{array}\right\} \Rightarrow \pi R^2 = 9\pi r^2 \Rightarrow R^2 = 9r^2$

Per tant $\left\{\begin{array}{l} R = r+10 \\ R = 3r \end{array}\right. \Rightarrow 3r = r+10 \Rightarrow 2r = 10 \Rightarrow \boxed{r = 5\, cm,\ R = 5+10 = 15\, cm}$

④ a) $\sqrt[3]{x} = -5 \Rightarrow x = (-5)^3 = \boxed{-125}$

b) Àrea $= 0{,}01\, m^2 \Rightarrow$ costat $= \sqrt{0{,}01\, m^2} = \boxed{0{,}1\, m}$

c) $1{,}08321 \cdot 10^{12}\, Km^3 = \frac{4\pi \cdot radi^3}{3} \Rightarrow radi^3 = \frac{3 \cdot 1{,}08321 \cdot 10^{12}}{4\pi} \Rightarrow$

$\Rightarrow radi = \sqrt[3]{\frac{3 \cdot 1{,}08321 \cdot 10^{12}}{4\pi}} = \sqrt[3]{\frac{3{,}24963}{4\pi}} \cdot 10^4 \simeq 0{,}6371006 \cdot 10^4\, Km$
$\simeq \boxed{6371{,}006\, Km}$

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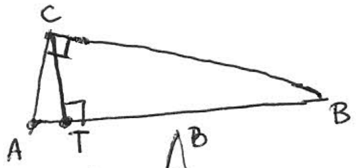
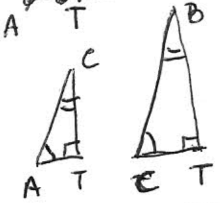
a)



Cal demostrar que $\frac{AT}{TC}=\frac{TC}{TB}$ (o bé $TC^2=AT\cdot TB$ o bé $TC=\sqrt{AT\cdot TB}$)

Per aconseguir-ho considerem els triangles $\widehat{ATC}$ i $\widehat{CTB}$:



→ Aquests triangles tenen els angles iguals (*) i, per tant, són semblants. I llavors els costats són proporcionals. Per tant $\boxed{\frac{AT}{CT}=\frac{CT}{BT}}$

(*) $\left.\begin{array}{l}\widehat{ACT}+\widehat{TCB}=90^\circ\\ \widehat{ACT}+\widehat{TAC}=90^\circ\end{array}\right\} \Rightarrow \widehat{TCB}=\widehat{TAC}=\alpha \Rightarrow \widehat{TCA}=90^\circ-\alpha=\widehat{TBC}$

b) $\sqrt{12}=\sqrt{4\cdot 3} \Rightarrow$ Es pot construir a partir d'un triangle rectangle d'hipotenusa $4+3=7$ i projeccions dels catets sobre la hipotenusa 4 i 3

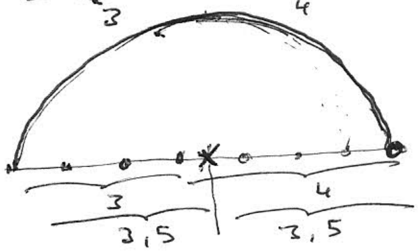
Etapes:

(1) Considerem una unitat de mesura →

(2) Construïm

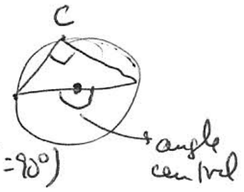
1. Segment de longitud 4+3 →

2. Semicercle de diàmetre 4+3



3. Perpendicular al diàmetre pel punt que parteix en 3+4

Solució



Nota: El triangle ABC és rectangle en C perquè el seu angle central mesura 180°. (I llavors $\hat{C}=90^\circ$)

Per tant, l'altura d'aquest triangle "és" $\sqrt{3\cdot4}=\sqrt{12}$

→ mesura

6

a) $\sqrt{5}\cdot\sqrt{125}=\sqrt{625}=\sqrt{5^4}=\boxed{5^2=25}$

b) $\sqrt{12}\cdot\sqrt[3]{12}=\sqrt[6]{12^3}\cdot\sqrt[6]{12^2}=\sqrt[6]{12^5}=\sqrt[6]{(2^2\cdot 3)^5}=\sqrt[6]{2^{10}\cdot 3^5}=\boxed{2\sqrt[6]{2^4\cdot 3^5}}\boxed{=2\sqrt[6]{3888}}$

c) $\sqrt{12}+\sqrt{27}=\sqrt{4\cdot 3}+\sqrt{9\cdot 3}=2\sqrt{3}+3\sqrt{3}=\boxed{5\sqrt{3}}$

d) $\frac{\sqrt{a}\;\sqrt[3]{b^2}}{\sqrt{a^3b}}=\frac{a^{1/2}\cdot b^{2/3}}{a^{3/2}\,b^{1/2}}=a^{1/2-3/2}\cdot b^{2/3-1/2}=a^{-1}\cdot b^{\frac{4-3}{6}}=\frac{b^{1/6}}{a}=\boxed{\frac{\sqrt[6]{b}}{a}}$