

Nom _____

1.- Resol aquest sistema emprant el mètode de Gauss (3 punts)

$$\begin{cases} 3x + 8y - 4z = -44 \\ 5x - 8y + 4z = 76 \\ -x + 7y - 7z = -81 \end{cases}$$

$$\left(\begin{array}{ccc|c} 3 & 8 & 4 & -44 \\ 5 & -8 & 4 & 76 \\ -1 & 7 & -7 & -81 \end{array} \right)$$

$$\left(\begin{array}{ccc|c} 3 & 8 & 4 & -44 \\ 0 & 64 & -32 & -448 \\ 0 & -29 & 25 & 287 \end{array} \right)$$

$$\left(\begin{array}{ccc|c} 3 & 8 & 4 & -44 \\ 0 & 64 & -32 & -448 \\ 0 & 0 & -672 & -5736 \end{array} \right)$$

$$z = \frac{-5736}{-672}; \quad \mathbf{z = 8}$$

$$64y - 32 \cdot 8 = -448; \quad 64y - 256 = -448; \quad 64y = -448 + 256; \quad 64y = -192; \quad \mathbf{y = -3}$$

$$-x + 7 \cdot (-2) - 7 \cdot 8 = -81; \quad -x - 21 - 56 = -81; \quad -x = -81 + 77; \quad -x = -4; \quad \mathbf{x = 4}$$

2.- Calcula la matriu X de manera que $A \cdot X = B$ (3 punts)

$$\begin{pmatrix} -3 & -8 \\ -1 & -2 \end{pmatrix} \cdot X = \begin{pmatrix} -3 & -26 \\ 1 & -8 \end{pmatrix}$$

$$\text{a.- } \begin{vmatrix} -3 & -8 \\ -1 & -2 \end{vmatrix} = 6 - 8 = -2$$

$$\text{b.- } A^T = \begin{pmatrix} -3 & -1 \\ -8 & -2 \end{pmatrix}$$

$$\text{c.- } \text{Adj } A^T = \begin{pmatrix} -2 & 8 \\ 1 & -3 \end{pmatrix}$$

$$\text{d.- } A^{-1} = -\frac{1}{2} \begin{pmatrix} -2 & 8 \\ 1 & -3 \end{pmatrix}$$

$$\text{e.- } A^{-1} \cdot B =$$

$$-\frac{1}{2} \begin{pmatrix} -2 & 8 \\ 1 & -3 \end{pmatrix} \cdot \begin{pmatrix} -3 & -26 \\ 1 & -8 \end{pmatrix} = -\frac{1}{2} \begin{pmatrix} 6 + 8 & 52 - 64 \\ -3 - 3 & -26 + 24 \end{pmatrix} = -\frac{1}{2} \begin{pmatrix} 14 & -12 \\ -6 & -2 \end{pmatrix} =$$

$$\begin{pmatrix} -7 & 6 \\ 3 & 1 \end{pmatrix}$$

3.- Resol el sistema de l'exercici 1 mitjançant la matriu inversa (determinant 1 punt, matriu adjunts 1 punt, matriu inversa 1 punt i matriu X 1 punt)

a.- Expressió del sistema com a producte de matrius

$$\begin{pmatrix} 3 & 8 & -4 \\ 5 & -8 & 4 \\ -1 & 7 & -7 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -44 \\ 76 \\ -81 \end{pmatrix}$$

b.- Determinant del sistema:

$$\begin{vmatrix} 3 & 8 & -4 \\ 5 & -8 & 4 \\ -1 & 7 & -7 \end{vmatrix} = 168 - 140 - 32 - (-32 + 84 - 280) = -4 - (-228) = 224$$

$$c.- A^T = \begin{pmatrix} 3 & 5 & -1 \\ 8 & -8 & 7 \\ -4 & 4 & -7 \end{pmatrix}$$

$$d.- \text{Adj } A^T = \begin{pmatrix} \begin{vmatrix} -8 & 7 \\ 4 & -7 \end{vmatrix} & -\begin{vmatrix} 8 & 7 \\ 4 & -7 \end{vmatrix} & \begin{vmatrix} 8 & -8 \\ 4 & -4 \end{vmatrix} \\ -\begin{vmatrix} 5 & -1 \\ 4 & -7 \end{vmatrix} & \begin{vmatrix} 3 & -1 \\ -4 & -7 \end{vmatrix} & -\begin{vmatrix} 3 & 5 \\ -4 & 4 \end{vmatrix} \\ \begin{vmatrix} 5 & -1 \\ -8 & 7 \end{vmatrix} & -\begin{vmatrix} 3 & -1 \\ 8 & 7 \end{vmatrix} & \begin{vmatrix} 3 & 5 \\ 8 & -8 \end{vmatrix} \end{pmatrix} = \begin{pmatrix} 28 & 28 & 0 \\ 31 & -25 & -32 \\ 27 & -29 & -64 \end{pmatrix}$$

$$e.- A^{-1} = \frac{1}{224} \begin{pmatrix} 28 & 28 & 0 \\ 31 & -25 & -32 \\ 27 & -29 & -64 \end{pmatrix}$$

$$f.- \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{224} \begin{pmatrix} 28 & 28 & 0 \\ 31 & -25 & -32 \\ 27 & -29 & -64 \end{pmatrix} \cdot \begin{pmatrix} -44 \\ 76 \\ -81 \end{pmatrix} = \frac{1}{224} \begin{pmatrix} 896 \\ -672 \\ 1792 \end{pmatrix} = \begin{pmatrix} 4 \\ -3 \\ 8 \end{pmatrix}$$