2.Graphs and functions.

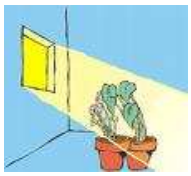
Definition of a function

- In everyday life, many quantities depend on one or more changing variables.

T.1. Work in pairs and match each picture with one sentence, then you'll report the results to the whole group.

1.

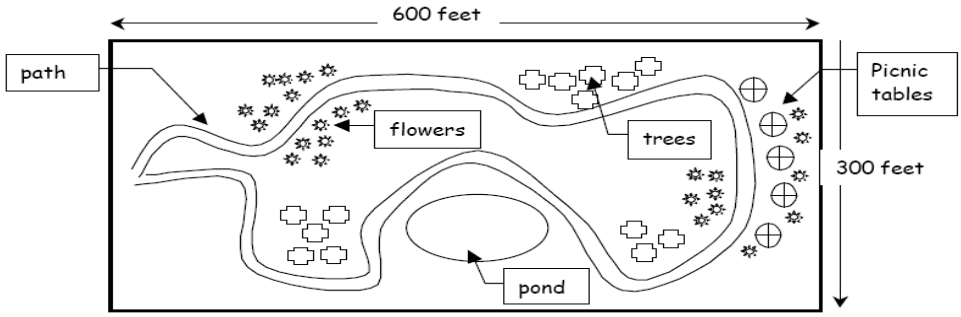
2.



3.

- Plant growth depends on sunlight and rainfall.
- Speed depends on distance travelled and time taken.
- Test marks depend on attitude, doing homework, and working everyday. (Among many other variables!!)

T.2. The Garden (Work in pairs, then we will report the results in the plenary)



The garden in the picture above is open every day from 10:00 a.m. to 5:00 p.m.

It costs 2.00 € per visitor to walk through the garden.

Which amounts are variables? Write a "V" next to them.

_____ 1. The area covered by the garden

_____ 2. The number of people who visit the garden each day

_____ 3. The amount of sunlight that falls on the garden each day

_____ 4. The number of hours the garden is open each day

_____ 5. The amount of water in the pond

_____ 6. The amount of plant food the gardeners use each week

_____ 7. How much money each person pays to visit the garden

_____ 8. The length of the path through the garden

_____ 9. The number of bees in the garden

_____ 10. The amount of rain that falls on the garden each day.

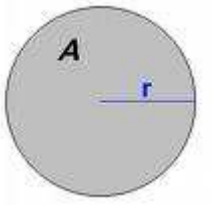
DEFINITION OF A FUNCTION:
A function is a rule that relates how one variable (x) depends on another variable (y).
And for each value of "x", there is EXACTLY ONE value of "y".

- Then we say: "The dependent variable (y) is a function of the independent variable (x)"
- The function notation: We normally write functions as…

F(x)

…and we read this as "function f of x".

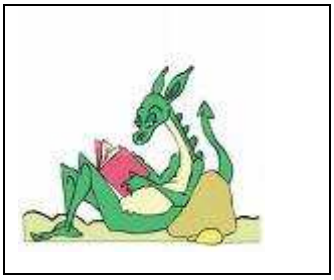
- Example: Do you remember the formula for calculating the area of a circle?



$A=\Pi x r^2$

T.3. Work in pairs and answer the following questions. Then we will report the results in the plenary class.	
Is there any constant? Which one/s?	
Is there any variable? Which one/s?	
What happens if the radius increases?	
What happens if the radius decreases?	
Calculate the area if the radius= 2cm	
Calculate the area if the radius= 3cm	
Calculate the area if the radius= 4cm	
Is this a function? Why?	
Which is the dependent variable? Why?	
Which is the independent variable? Why?	
Complete the sentence: The "……" is a function of the "……"	

Ways of expressing a function



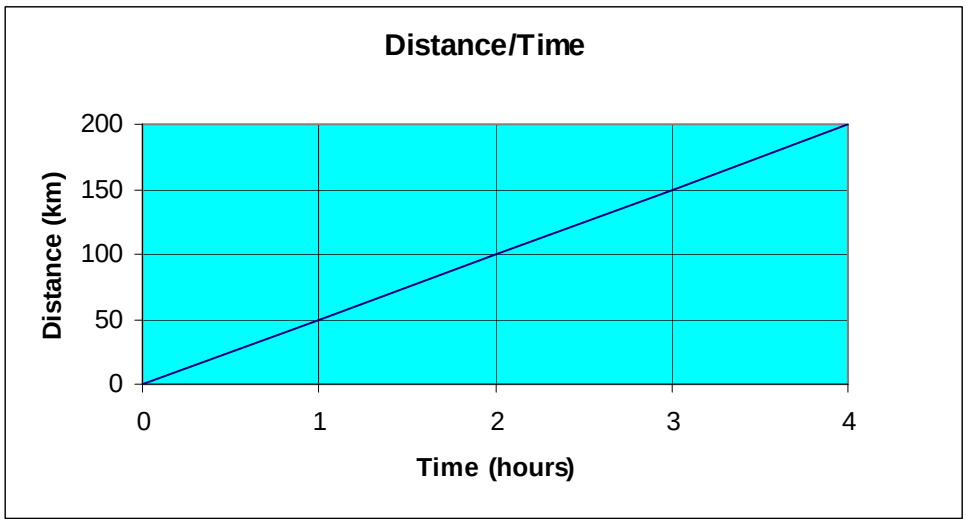
- A SENTENCE OR A TEXT:

A car travels at a steady speed of 50 km/h, along a motorway

- A TABLE OF VALUES

Time (hours)	0	1	2	3	4
Distance (km)	0	50	100	150	200

- A GRAPH. A visual representation of data that displays the relationship among variables.



- What does the title tell you?
- Which is the dependent variable?
- Which is the independent variable?
- What units are used on the "x" and "y" axes?
- What is the scale on the y-axis? And on the x-axis?
- How far did the car travel each hour?
- What's the distance travelled after 1 hour?
- What's the distance travelled after 2 hours?
- What's the distance travelled after 3 hours?
- What happens when the time increases?

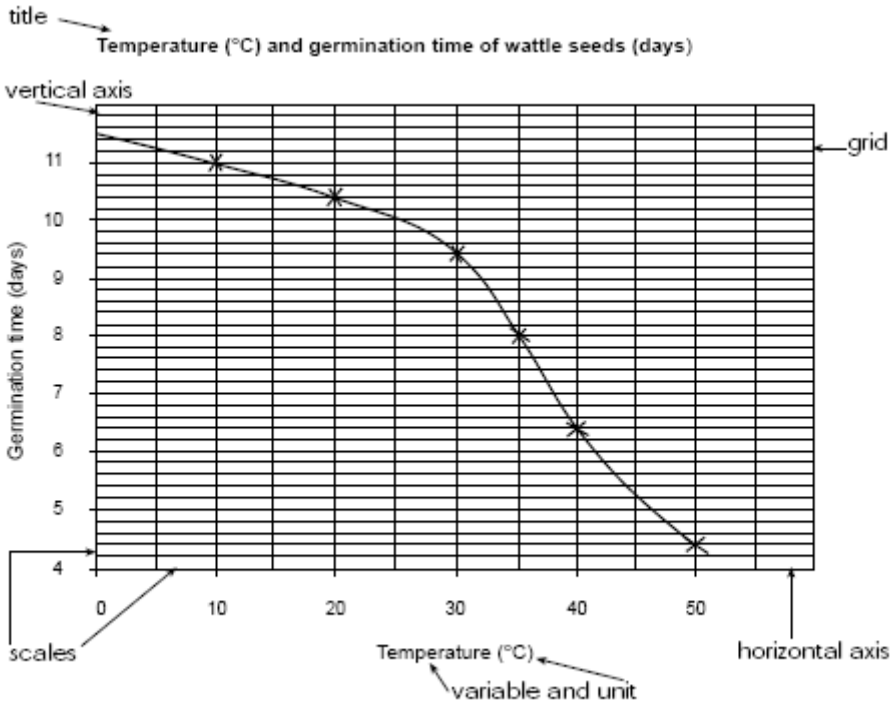
- An algebraic expression: a group of numbers, symbols and variables (letters) that express an operation or a series of operations (not all the function can be written as an algebraic expression)

Distance=speed·time	y=50·x

Interpreting graphs

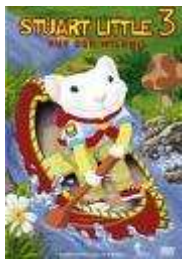
WHAT ARE SOME FEATURES OR CHARACTERISTICS OF A GRAPH?

- A TITLE that describes what the graph shows.
- A GRID that is used to plot points or other data.
- A HORIZONTAL AXIS or X-axis that is labelled with the name of a variable and the units represented (the independent variable).
- A VERTICAL AXIS or Y-axis that is labelled with the name of a variable and the units represented. (the dependent variable).



EXAMPLE:

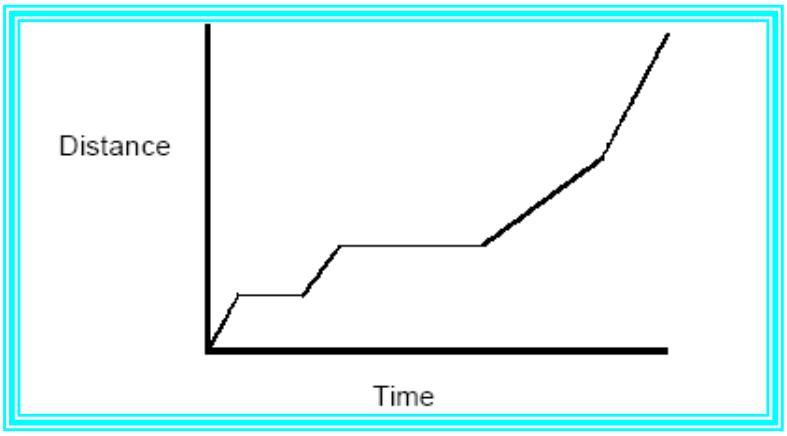
In the book *Stuart Little* by E. B. White, the chapter titled "The Sailboat Race" tells what happened to Stuart during a boat race on a windy day.



His race was interrupted first by a huge wave that turned his boat over.

And again later when he sails into a huge paper bag and could not get out for an extended period of time.

This adventure might be represented by the following graph:



T.4. Match each sentence with the correct part of the graph. (pairs)
1) He starts racing
2) The huge wave turns his boat over
3) He races again
4) He sails into a huge paper bag
5) He gets out of the bag and and goes on racing

Drawing a function

We are going to plot the graph of the function: y=x +2.

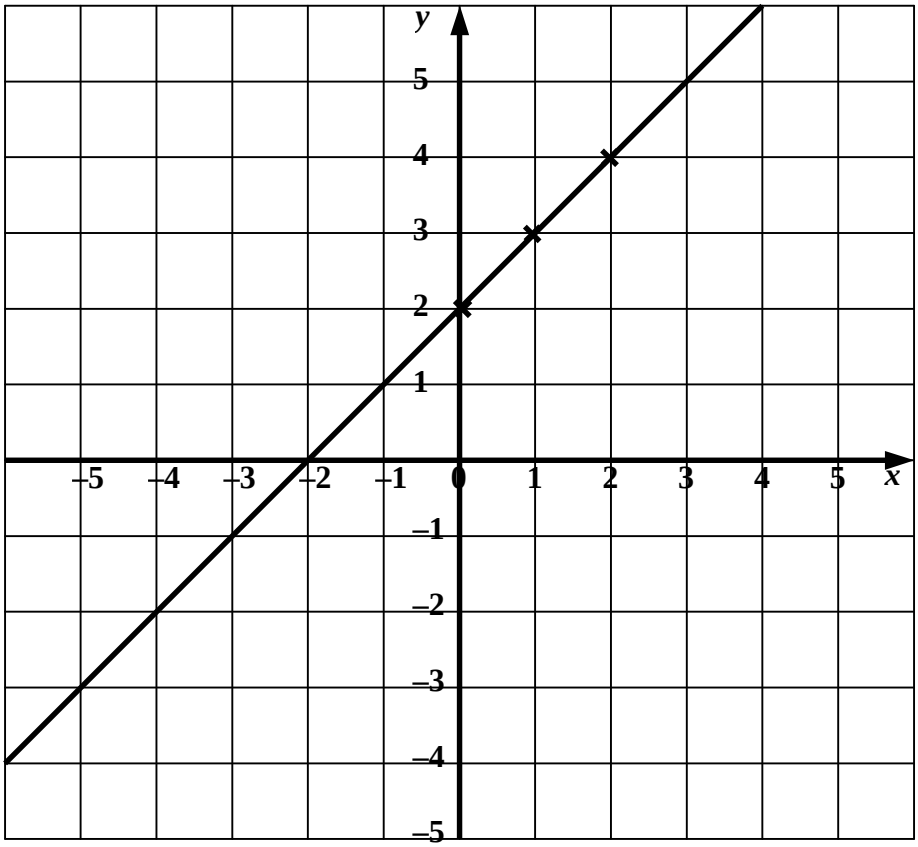
- First of all, we need a table of values.

- Choose these values for x: 0, 1, 2, -1, -2.
- Work out the corresponding values for y.

X	y=x+2	coordinate
0	Y=0+2=2	(0,2)
1	Y=1+2=3	(1,3)
2	Y=2+2=4	(2,4)
-1	Y=(-1)+2=1	(-1,1)
-2	Y=(-2)+2=0	(-2,0)

- Now, we can plot the graph.

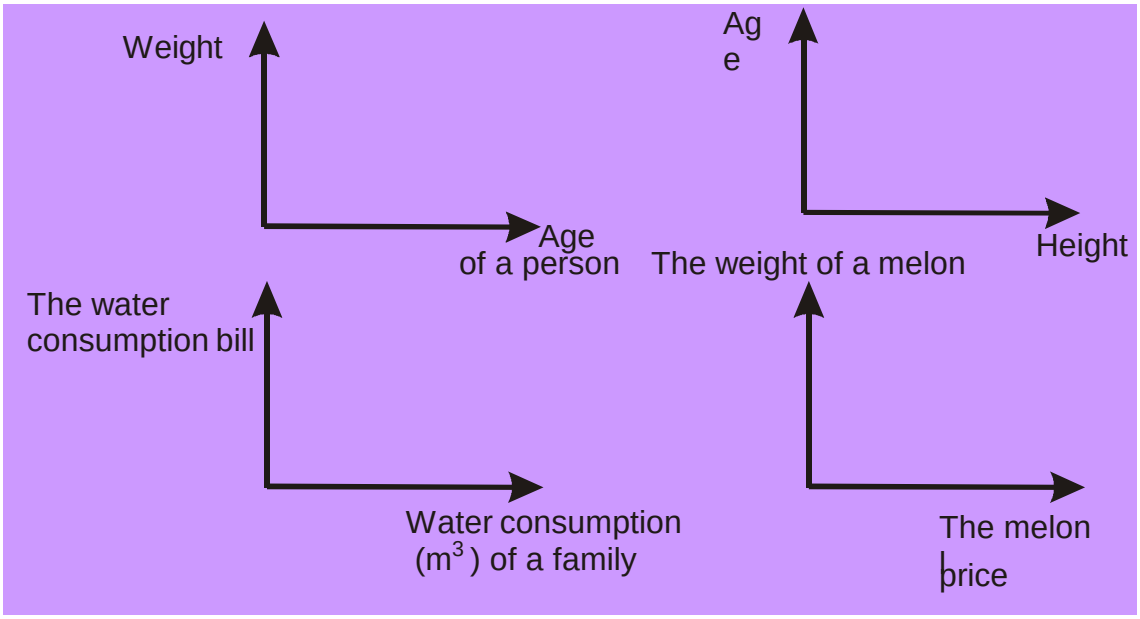
- Draw the x-axis and y-axis, and numerate them.
- Plot the points.
- Join the points with a straight line.
- Label the line with its equation.



$y = x + 2$

T.5. Now it's your turn: Plot the graph of the function: $y = x^2$

- ACTIVITY 1. IS THERE ANY RELATIONSHIP BETWEEN THESE VARIABLES? Work in groups, and we will report the result in the plenary.



There is a relationship between…and…	because/as/since	if the "…" increases, the "…" increases/decreases
There isn't any relationship between…and…	because/as/since	if the "…"increases, the "…" doesn't increase/decrease
The "…" is function of the "…"		

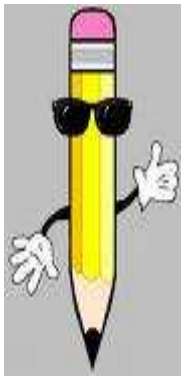
- ACTIVITY 2. **VERTICAL LINE TEST FOR A FUNCTION**

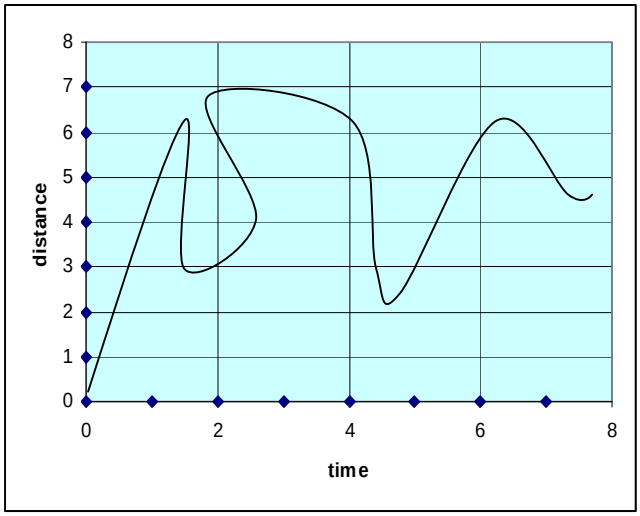
If any **vertical line** passes through **no more than one point** of the graph of a relation, then the relation is a function.

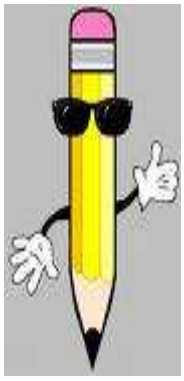
Look at the graphs below. To do the test,

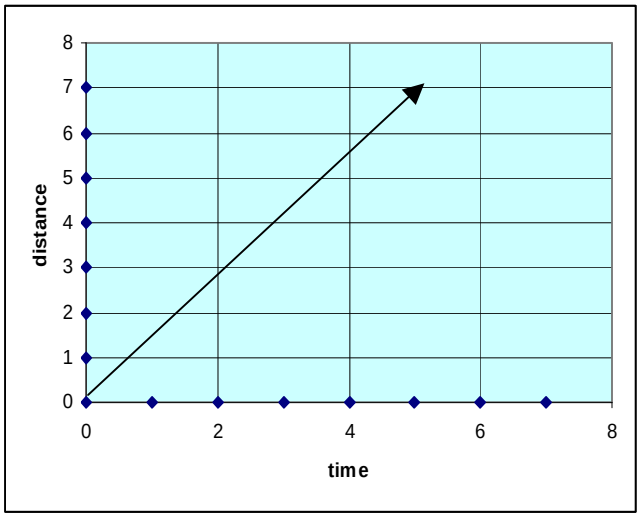
- Place a pencil to the left of the graph to represent a vertical line.
- Move it to the right across the graph.
- The graph is showing a function, if for each value of x, this vertical line passes through exactly **one point on the graph**.
- Which of the following graphs is the graph of a function?
- Work in pairs and then we'll report the results in the plenary.

The first/second one	is a function	because for each value of "x" there is/are/isn't/ aren't…
The first/second one	is not a function	because for each value of "x" there is/are…

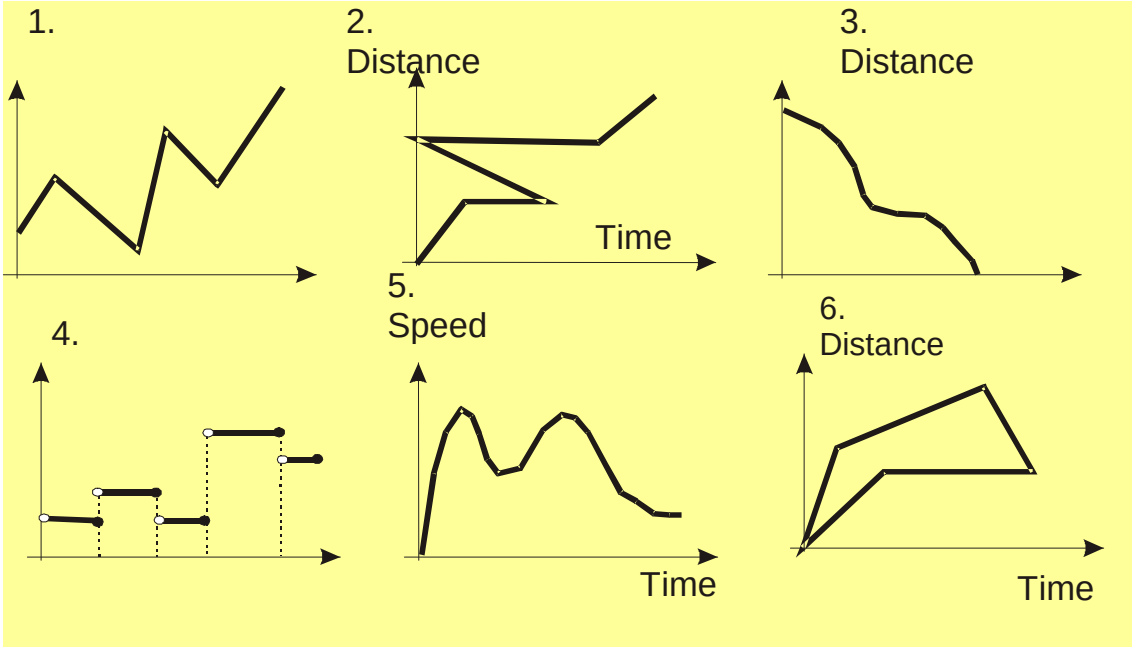



distance
time
0
1
2
3
4
5
6
7
8
0
2
4
6
8




distance
time
0
1
2
3
4
5
6
7
8
0
2
4
6
8

- ACTIVITY 3. Use the vertical line test to determine if each relation is a function. Work in pairs and later, plenary.



The first one The second one The third one…	is a function is not a function	because	for each value of "x"	there is… there are…

- ACTIVIY 4.

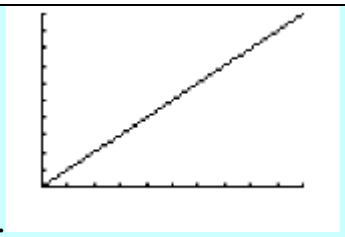
The fixed cost for a company to operate a certain plant is $3,000 (electricity, gas, water…) per day. It also costs $4 for each unit produced in the plant. Express the daily cost "*C*" of operating the plant as a function of the number "*n*" of units produced. Work in pairs and then we will report the results in the plenary class.

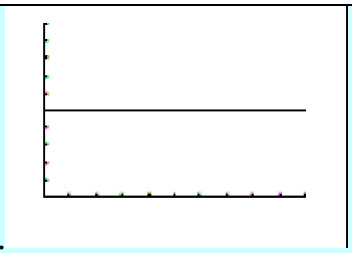
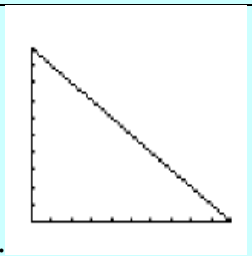
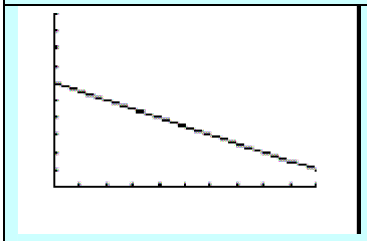
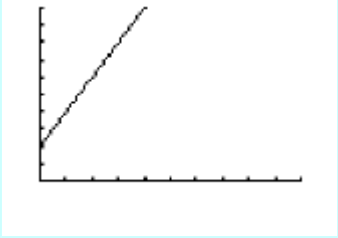
- ACTIVIY 5. An architect designs a window with the shape of a rectangle .The base of the window is 10 cm less than the height. Express the perimeter "*p*" of the window as a function of the height of the window. Work in pairs and then we will report the results in the plenary class.

X cm.

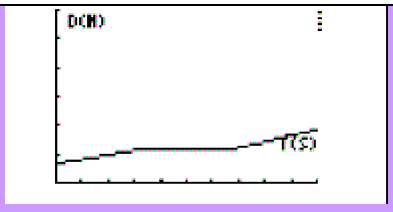
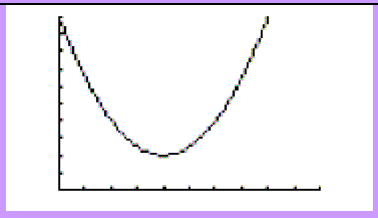
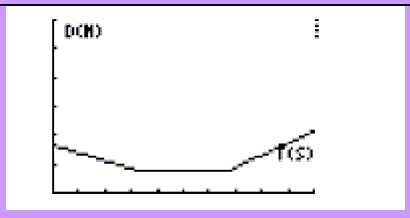
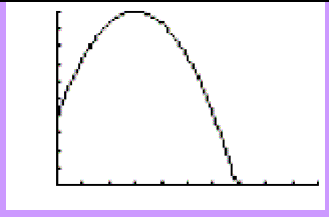
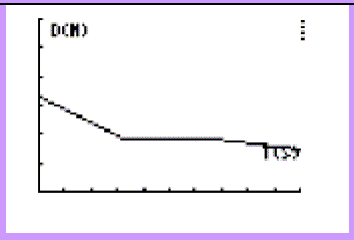
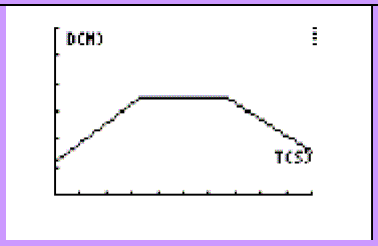
(X-10) cm.

- ACTIVITY 6. Match each graph with a movement (you are in front of the teacher's chair). Work in pairs and then we will report the results in the plenary class.

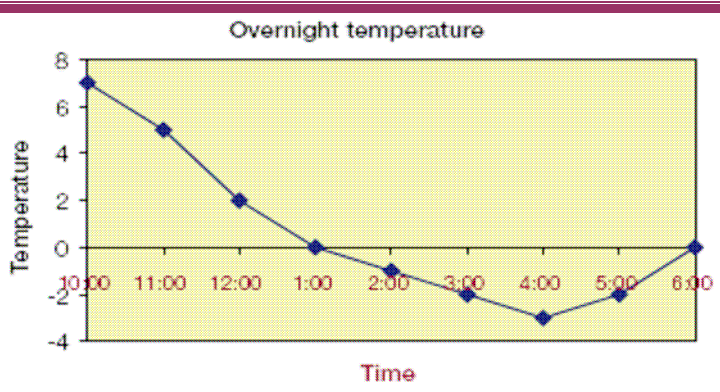
1. 	a)Begin 2 metres away from the teacher's chair and walk briskly at a steady pace away from the teacher's chair.

2. 	b)Begin about 5 metres away from the teacher's chair and stand still
3. 	c)Begin right in front of the teacher's chair and walk at a moderate but steady pace away from the teacher's chair.
4. 	d)Begin right in front of teacher's chair and walk briskly away from teacher's chair, turn round, and walk briskly back to where you began.
5. 	e) Begin about 6 metres away from the teacher's chair and walk at a slow but steady pace towards the teacher's chair.
6. 	f) Begin about 10 metres away from the teacher's chair and walk at a moderate but steady pace towards the teacher's chair.

- ACTIVITY 7. Match each graph with a story.

a)While Mikey was standing on the beach, he threw a rock as high as he could before the rock landed in the water.	1. 
b) David sprinted for twenty seconds. He stopped for thirty seconds to Catch his breath. He then continued sprinting for twenty more seconds.	2. 
c) During a recent thunderstorm, the water level in the river increased at a steady rate and remained at a higher level for a period of time. The water then decreased nearly to the original level.	3. 
d) The pilot decreased the plane's altitude to adjust for turbulence. Once the turbulence was over the pilot returned the aircraft to its previous cruising altitude.	4. 
e) Diane threw her paper airplane. The airplane dropped at a constant speed, levelled off, and then rose at a constant speed until it was caught by Diane's friend Ellen.	5. 
f) Attendance at the library's summer reading program was good at first, but it decreased steadily and then levelled off as families began their summer holidays.	6. 

- ACTIVITY 8. Look at the 'Overnight temperature' graph for a desert area showing outside temperatures in °C, and answer the questions (work in pairs and then we will report the results in the plenary).



- What were the highest and lowest temperatures? When did these occur?
- When was the temperature at freezing point (0 °C)?
- How far did the temperature fall from 11:00 pm to 3:00 am?
- How long did the temperature stay below zero?
- What temperature was it at 11:30 pm?
- Use the Internet to research temperatures in different parts of the world.

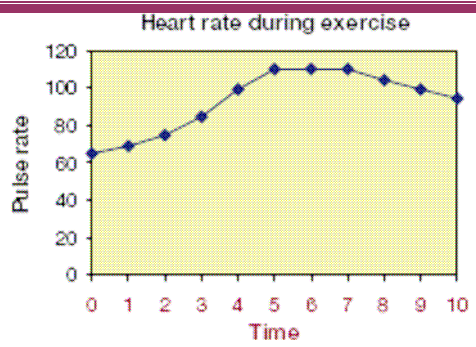
DID YOU KNOW?

Forgotten People: The Saharawis of Western Sahara

- Imagine being separated from your family and your home by a foreign invasion.
- Imagine living in a refugee camp in one of the most severe **desert** environments on earth.
- Imagine having your time in exile extended to an entire generation.
- Imagine, finally, enduring this situation with precious little solidarity and attention from the wider world.

If you want to learn more about "Saharawis", visit:
http://www.refugeesinternational.org/content/article/detail/869/

- ACTIVITY 9. Look at the graph below, and answer the questions (work in pairs and then we will report the results in the plenary).



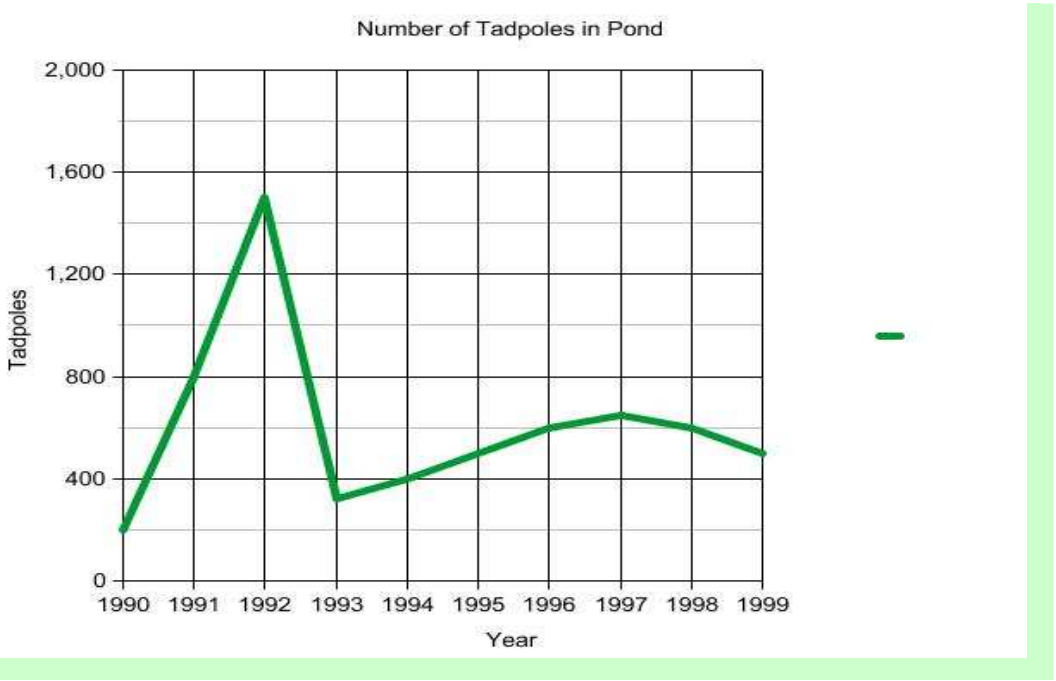
- How long did it take for the pulse rate to reach its highest level?
- What was the pulse rate at the start of the experiment?
- What was the pulse rate likely to have been after $2\frac{1}{2}$ minutes?
- When do you think the person stopped exercising?
- When was the heart rate increasing most?
- Carry out your own exercise experiment together and draw a line graph showing your heart rates. How do they compare?

The results are similar because…
The results are different because…

- ACTIVITY 10. The verbs in the box can be used to describe changes seen on graphs. Answer the questions (work in pairs, and then we'll report the results in the plenary).

PEAK	RISE	SOAR	FLUCTUATE	DECLINE	DROP	CLIMB	INCREASE

1. Circle the verbs that mean to go up.
2. Underline the verbs that mean to go down.
3. Put a star next to the verb that means to go up and down.
4. Use an arrow to indicate the word that means to reach its highest level.



- Use the words in the box above to help you complete the sentences.

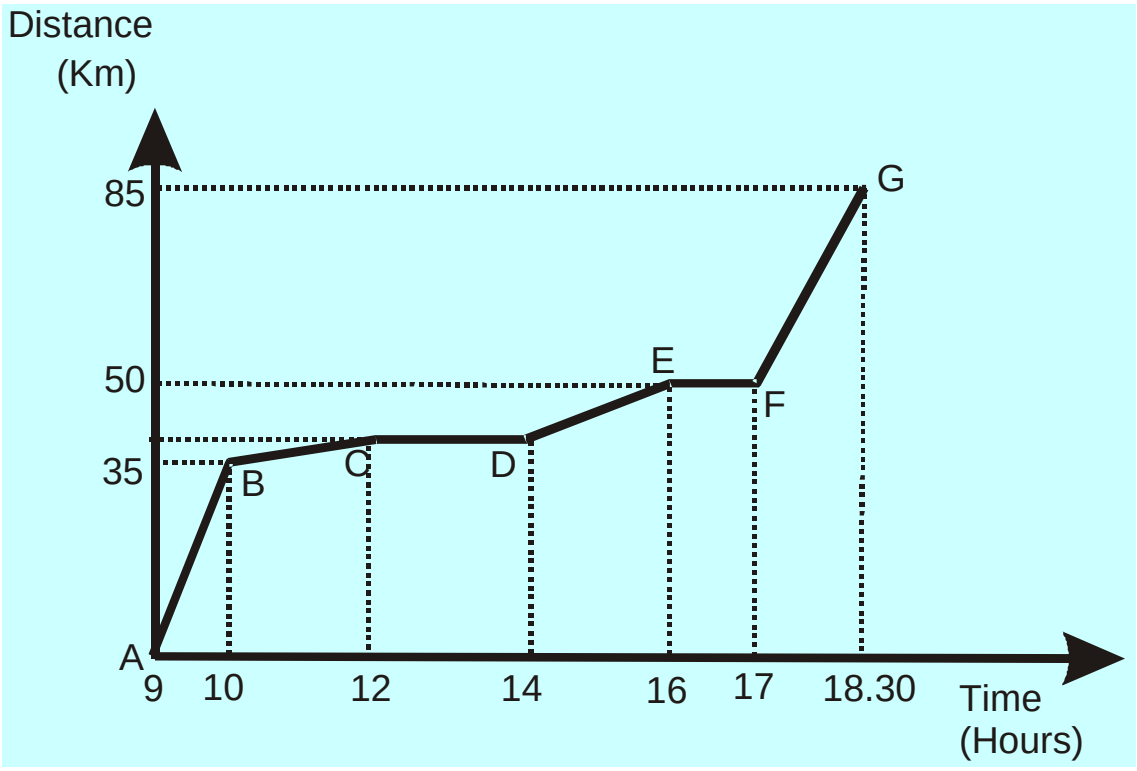
1. In the year 1990, the tadpole populations began to _____ rapidly.
2. The tadpole population reached a _____ in 1992.
3. Between 1992 and 1993, the population of tadpoles _____
4. The tadpole population _____ after 1993.
5. Between 1995 and 1999, population _____
6. How many tadpoles were in the pond at its highest point?

_____

7. How many tadpoles were present in the pond in 1998? _____
8. Between 1998 and 1999, the tadpole population _____

- ACTIVITY 11. A TRIP TO THE COUNTRYSIDE

  (Answer the following questions, work in pairs and later plenary). Maria, Luis, and José went on a trip to the countryside, as the following graph shows:



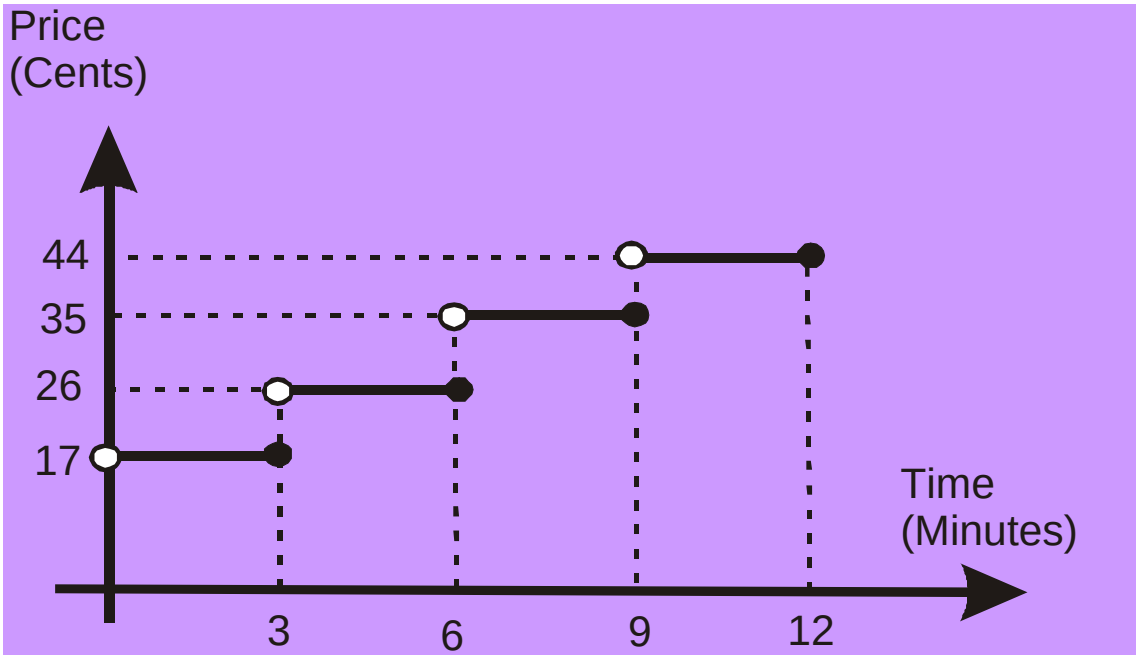
- Which is the independent variable?
- Which is the dependent variable?
- Is this function continuous or discontinuous?

The graph of a **continuous function** can be drawn **without lifting the pencil from the paper.**

- How many kms did they walk?
- When did they stop to have lunch and relax?
- Between which points did they walk faster? And more slowly?
- Draw a table of values to represent the data.
- Write a story about the trip.

Last week, they decided to go to….	They walked for…hours
They talked about…	They brought……for lunch
They saw a dog, a cat…	They stopped at an old Church…
As they were tired they stopped at…..for…hours.	Luis and Maria got angry because…. They were happy because…
They arrived at a wonderful lake, cottage, river…	They swam, they took a rest, they played football…
In the afternoon they stopped for…hours and they…	They came back at…..o'clock….because…

- ACTIVITY 12. This graph shows the relation between the price of a phone call within the city, and the time you can talk. (Work in pairs, later plenary).



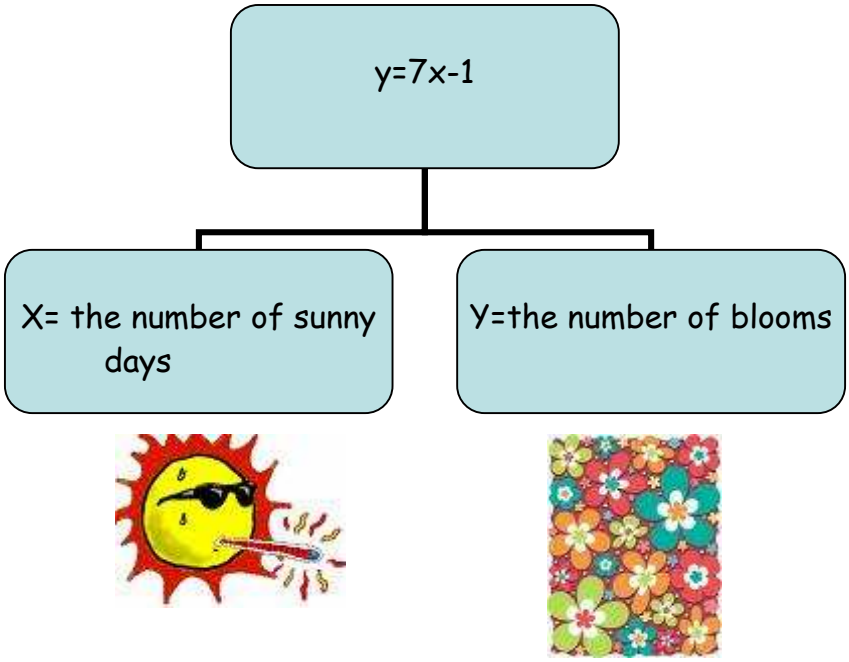
- Which is the dependent variable?
- Which is the independent variable?
- If you have got € 0,30, how long can you talk on the phone?
- How much does a 10 minute call cost?
- Is this function continuous or discontinuous?

- ACTIVITY 13. Steam in a boiler was heated to 150° C. Its temperature was then recorded each minute as follows:

Time (min)	0.0	1.0	2.0	3.0	4.0	5.0
Temp (°C)	150.0	142.8	138.5	135.2	132.7	130.8

Plot the graph.

- ACTIVITY 14. Plants. The number of blooms on a cactus is related to the number of days of sun it gets in a month. This relation is given by the equation y=7x-1.



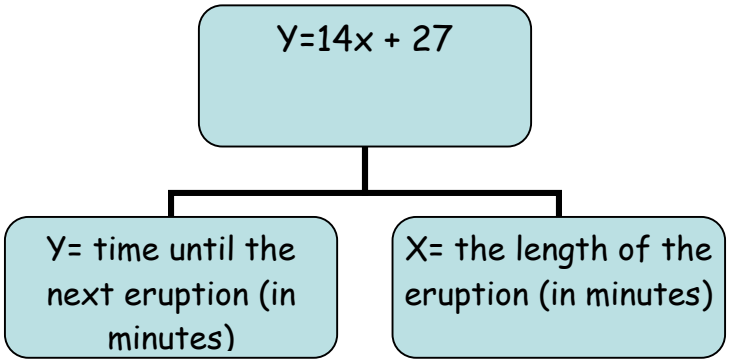
Working in pars (and later we'll comment on the results in the plenary):

A) Graph the equation.

B) Use the graph to determine the number of blooms on a cactus if it gets two days of sun a month.

- ACTIVIY 15. OLD FAITHFUL is a famous geyser.

The equation y=14x+27 can be used to predict its eruptions.



Suppose Old Faithful erupts at 9:46 a.m. for 3,4 minutes. At about what time will the next eruption occur? Use the graph to solve the problem. Then you can verify the answer by using the equation.

- ACTIVITY 16. Read these texts.

Geysers are rare. There are about 500 of them in Yellowstone National Park in the western United States, about 200 on the Kamchatka Peninsula in Russia, about 40 in New Zealand, 16 in Iceland, and another 50 scattered throughout the world in many other volcanic areas. Perhaps the most famous geyser is Old Faithful in Yellowstone National Park. It spouts a column of boiling water and steam to a height of about 30 to 55 metres on an approximately 90-minute timetable.

Kamchatka Peninsula (Russia)

Iceland

New Zealand

http://www.britannica.com/eb/article-9036646/geyser

KAMCHATKA (RUSSIA)

http://www.kamchatkapeninsula.com/

Kamchatka is a peninsula comparable in size to Japan. Its volcanic belt is made up of 29 active craters. It contains more than half of the world's Steller's Sea Eagles, as well as the largest population of brown bears.




The waters around Kamchatka are inhabited by the rare grey whale and approximately 300,000 seals and sea lions.

In the centre of Kamchatka is found Eurasia's only Geyser Valley. More than 150 thermal springs are scattered throughout the peninsula.

There aren't many people living there: less than 1 person per square kilometre. Most of the inhabitants live in the capital (Petropavlovsk).

But Kamchatka has got pollution problems, much of it because of the military presence on the Peninsula (Missiles have been dropped in this area).

The local economy in Kamchatka is severely depressed (mining and logging). By supporting sustainable economic development in Kamchatka, you can help locals feed their families without having to destroy their environment.

One way of doing this is to support eco-tourism on the peninsula. By visiting Kamchatka with an environmentally-responsible tour company, you help the local economy while making environmental protection an economic priority.

QUESTIONS

- What's a geyser?
- Have you ever seen a geyser?
- Who's the most famous brown bear that lives in Yellowstone Park?
- How many species can you find in Kamchatka?
- Which one do you like the most? Why?
- What do Kamchatka people do for a living?
- Why is there pollution in Kamchatka?
- How can we help them to save the environment?
- Would you like to visit Kamchatka? Why?

ACTIVIY 17. FINAL ACTIVITY.

a) Join these heads and tails:

A function is…	…a function of the independent variable
The dependent variable is…	…a group of numbers, symbols and variables that express an operation or a series of operations
A graph is…	…hasn't got a fixed value
An algebraic expression is…	…a rule that relates how one variable depends on another variable
A function can be expressed as…	…in the x-axis
The age of a person is…	…a function of his/her height
The weight of a person is…	…a visual representation of data that displays the relationship among variables
A constant	…just a value of Y
The independent variable is…	...in the y-axis.
In a function, for each value of x there is…	..a function of his/her age
A grid is…	…a text, a table of values, a graph and an equation.
The weight of a person is…	…has got a fixed value
The dependent variable is…	…a function of his/her weight
A variable…	…a pattern of regularly spaced horizontal and vertical lines forming squares

b) We are going to divide the class in groups of three. Each group has to prepare the questions related to the chart above. For example:

▪ What's a function?
▪ What's a visual representation of data…?

- In turns, the first group asks a question to the next group. If the answer is right, the group gets 5 points. Then the second group asks another question to the following group, and so on. The winner is the group with the maximum score.
- If the question is not correct, 5 points will be subtracted from the total number of points.
- If the answer is not correct, 5 points will be subtracted from the total number of points.

What's…..?	What's the definition of…?
Can you define…?	Where…?
Can you describe…?	Is it true that…?

c) We are going to divide the class in groups of three. Each group has to prepare a power-point presentation, summarizing the main points of the lesson (concepts and definitions, giving examples…). Your presentation will be recorded on the power point.