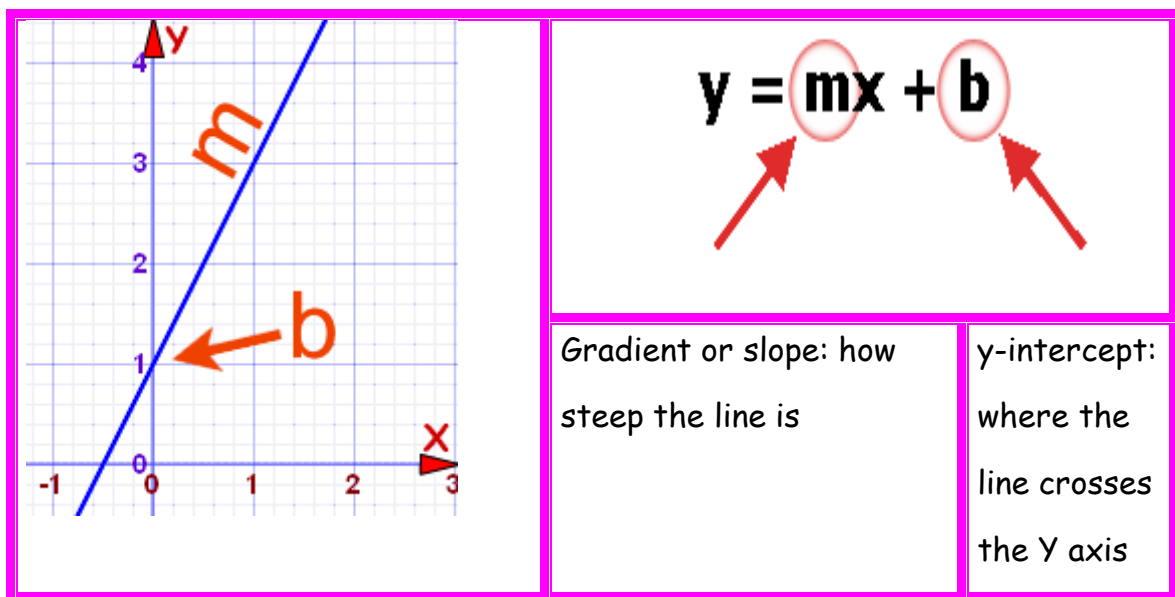# 3. Linear functions. Direct variation functions. Constant functions.

Linear functions

- In a linear function: **y varies linearly with x.**

- The graph of a linear function is a **straight-line** (but neither horizontal nor vertical)

- The general equation of a straight line is given in the form:

$$y = mx + b$$

| Gradient or slope: how steep the line is	y-intercept: where the line crosses the Y axis

Note: there are different types of "standard" formats for straight lines. This is called the slope-intercept form because "*m*" is the slope and "*b*" gives the *y*-intercept.

1

- The x-intercept, or horizontal intercept, is the x-coordinate of the point where the graph meets the horizontal axis (Where the line crosses the Y axis).

If y=0	➡	X? Solve for x	➡	The point is (?,0)
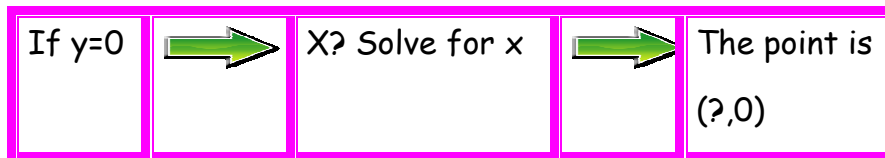
- The y-intercept, or vertical intercept, is the y-coordinate of the point where the graph meets the vertical axis (the point at which the line crosses the y-axis).

If x=0	➡	Y? Solve for y	➡	The point is (0, ?)

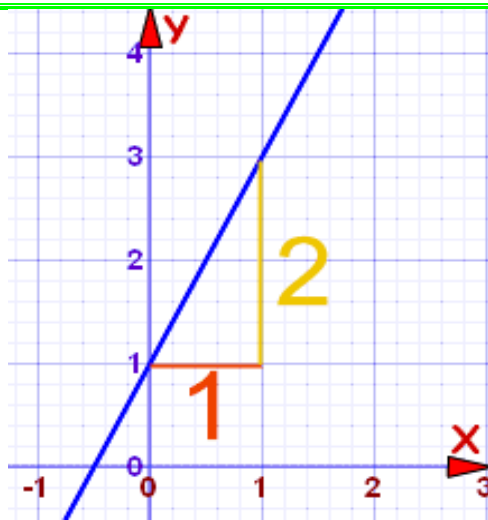|	The Y intercept is equal to 1.
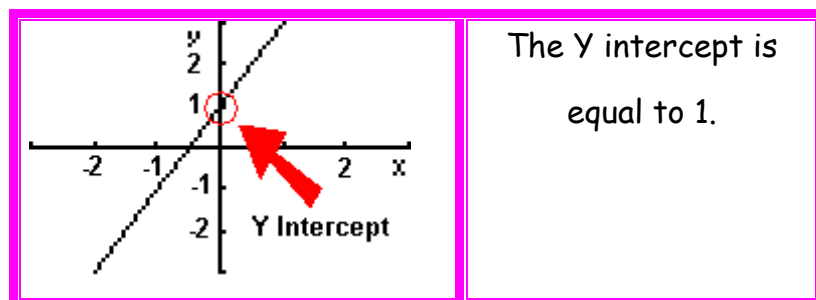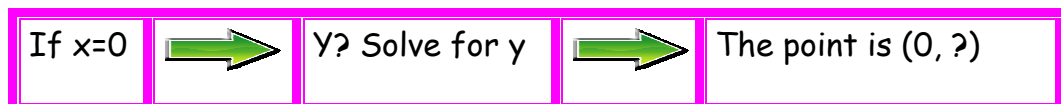
- The slope of the line is a measure of its inclination or steepness. The slope is used to tell us how much one variable (y) changes in relation to the change in another variable (x).

Rise   Change in y (vertical change)

GRADIENT= ----- = --------------------------------
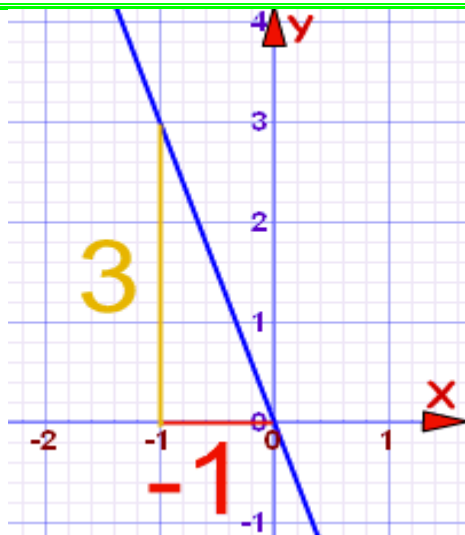
Run   Change in x (horizontal change)

T.1. Work out the equation of a straight line. Work in pairs and later plenary.

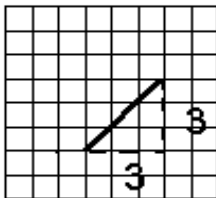|	
	m=gradient:
	y-intercept:
	Equation:
	m=gradient:
	y-intercept:
	Equation:

GRADIENT OF A STRAIGHT LINE

- Remember:

  - Starting from the left end of the line going across to the right is positive. But going across to the left is negative.

  - And up is positive, but down is negative.

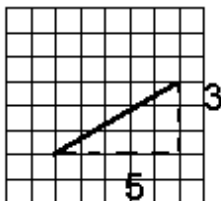| T.2. Try to find the gradient or slope. Work in pairs, and we will report the results in the plenary.	
Increasing linear function 	Gradient m=3/3 m=1
Increasing linear function 	Gradient m=

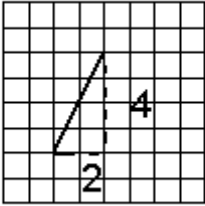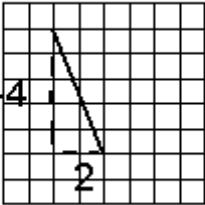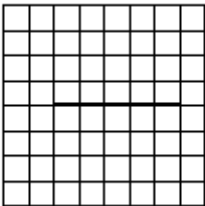| Increasing linear function	Gradient m=
Decreasing linear function	Gradient m=
Constant function	Gradient m=

T.3. CONCLUSION: Choose the correct answer (Work in pairs and later plenary.)
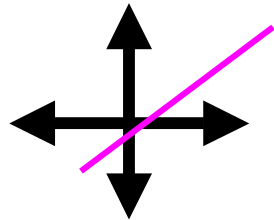
	graphs of increasing linear functions have <u>positive/negative</u> slope
	decreasing linear functions have <u>positive/ negative</u> slope
	Constant linear functions have slope…
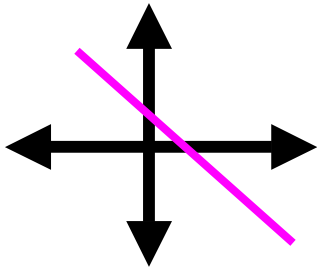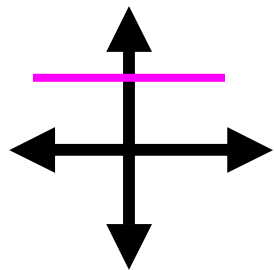
Constant functions

A constant function is a special linear function in which the

dependent variable is constant (a number):

The gradient or slope = 0

The y-intercept=b=constant
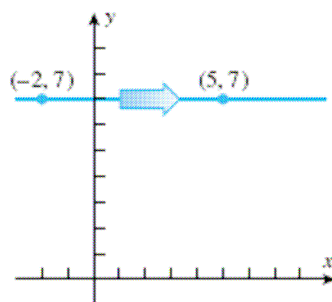
	A constant function has got the form:
$m = 0$ Traveling left to right, a point on the line neither rises nor falls.	Y=0x+b Y= b
	b=constant m=0. There is no inclination
	.

Direct variation functions

- The linear function, **y** = **mx**, where b is zero, is called a **direct variation function**.

Y=kx	K=gradient or slope= the constant of variation
	b= y-intercept=0

- We usually write y=kx, where k≠ 0. K is called the constant of variation.

- The graph of a direct variation **passes through the origin (0,0)**.

- We say that y varies directly as x. (y is directly proportional to x).

- Using a direct variation graph is one way to solve proportions.
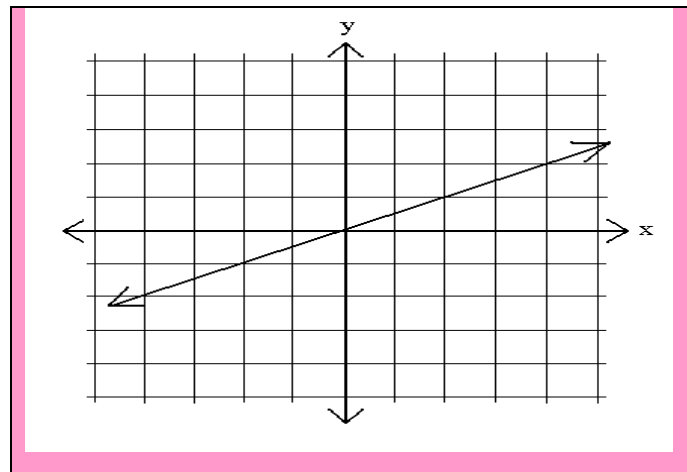
Activities

- ACTIVIY 1. Match the graphs below with the following equations.

  Work in pairs and then we will report the results in the plenary.

5=y	-3x+4=y	y=x/2	x=-3

(I)

(II)

(III)

(IV)
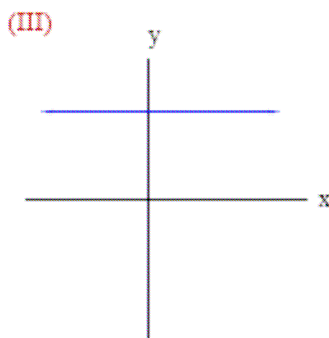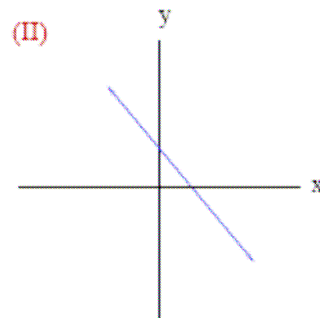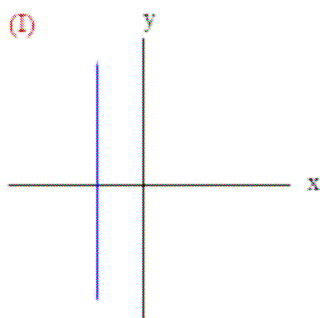
The graph number I corresponds to the equation…because…	the gradient is positive/negative the function is increasing/decreasing the y-intercept is… the x-intercept is… the value of the gradient is…

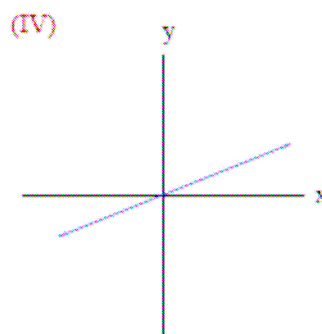- ACTIVIY2. We will divide the class in groups of two (then we will report the results in the plenary). You have to invent an equation of the form:

"y=mx +c", and you have to draw a diagram like the following one:

x intercept is …

$y = 0$

Sketch …

y intercept is …

$x = 0$

$y =$

Another point on the line is…
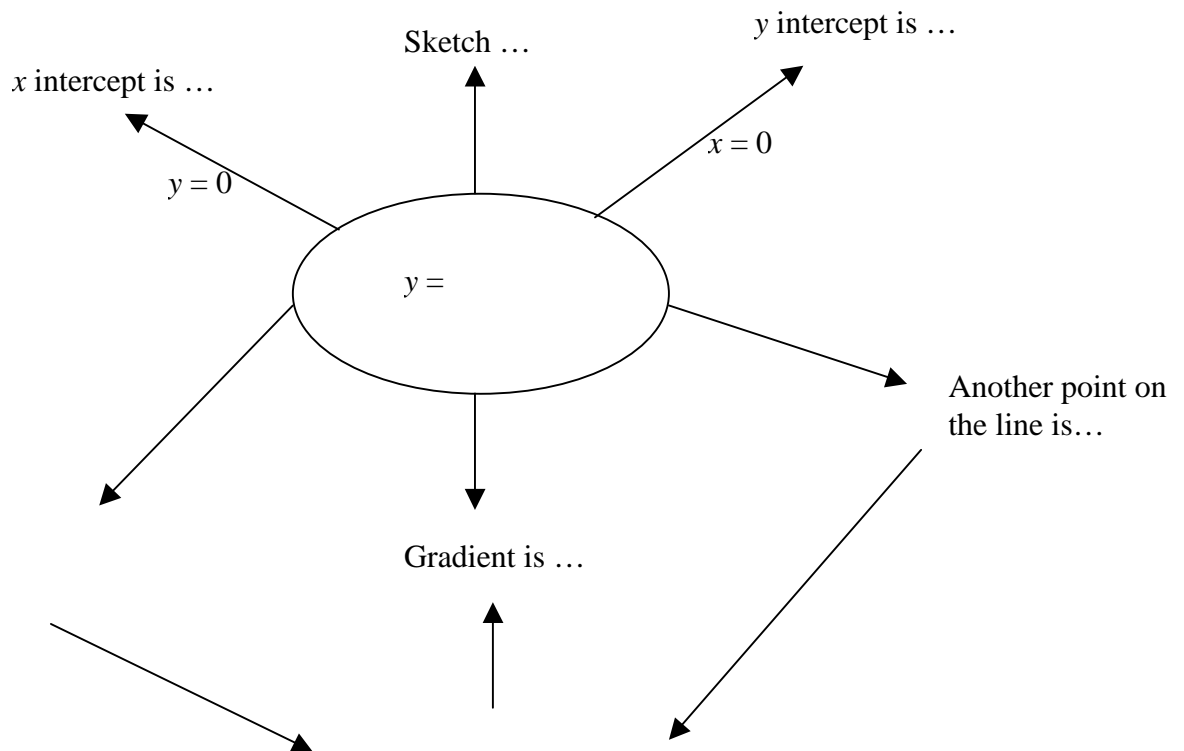
Gradient is …

A point of the line is……because if I substitute both values of x and y the result is…
It passes through the origin because…
The gradient is….It means that if x varies….units, then y varies…
The x intercept is…because it comes from the substitution of y by…
The y intercept is…because it comes from the substitution of x by…

- ACTIVIY 3. I am going to give you some cards with equations of linear functions, and you will have to classify them, working in pairs:

  a) Cards with the same y-intercept. Now write an equation that belongs to this category, and justify your choice.

  b) Cards with the same x-intercept. Now write an equation that belongs to this category, and justify your choice.

  c) Cards with the same slope. Now write an equation that belongs to this category, and justify your choice. What happens if 2 straight lines have the same slope?

- ACTIVIY 4. Work in pairs and then we will report the results in the plenary.

$950 $600 $60

  - James is saving his money to buy a Play station. He will need $950 to buy the play station, accessories, and a few games. He has $600 saved now, and he can put $60 into his savings at the end of each month.

  - Since the amount of money saved depends on how many months have passed, choose time, in months, as your independent variable and place it on the horizontal axis. Let "t" represent

the number of months passed, and make a mark for every month.

- Choose money saved, in dollars, as your dependent variable and place it on the vertical axis. Let "A" (A for amount) represent the amount saved in dollars. Since James saves $ 60each month, it will be convenient to let each box represent $60.

Amount saved A (dollars)

Time t (months)
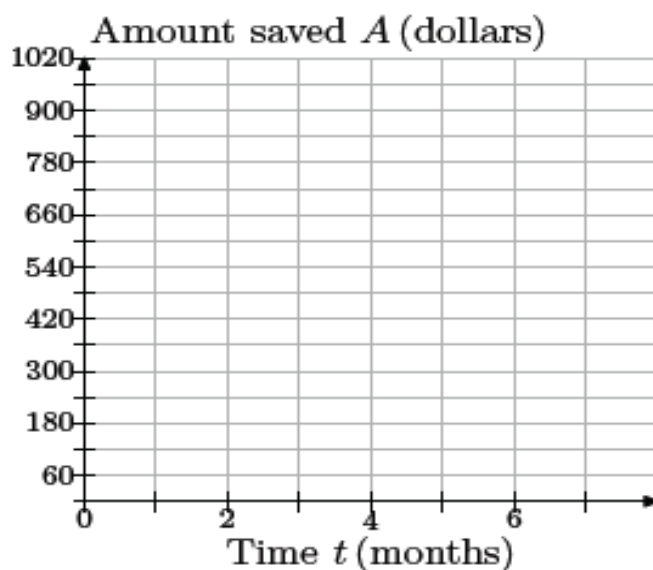
a) At month 0, James has $600 saved. This corresponds to the point (0, 600). Plot this point on your coordinate system.

b) For the next month, he saved $60more. Beginning at point (0, 600), move 1 month to the right and $60up and plot a new data point. What are the coordinates of this point?

c) Each time you go right 1 month, you must go up by $60 and plot a new data point. Repeat this process until you reach the edge of the coordinate system.

d) Draw a line through your data points.

e) Use your graph to estimate how much money James will have saved after 7 months.

After 7 months he will…

f) Using your graph, estimate how many months it will take him to have saved up enough money to buy his game station, accessories, and games.

It will take him……months to save the money to buy the play station…

g) Try to write the equation of the straight line.

- ACTIVIY 5. Work in pairs and later we will report the results in the plenary.

50 acorns until hibernation

8 acorns every 2 days
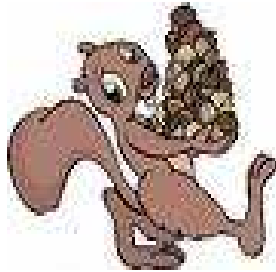
Earl the squirrel has only ten more days until hibernation. He needs to save 50 more acorns. He is tired of collecting acorns and so he is only able to collect 8 acorns every 2 days.

a) Let "t"represent time in days and make it your independent variable. Let "N"represent the number of acorns collected and make it your dependent variable. Draw a scaled coordinate system on a sheet of graph paper.

b) At time $t = 0$, Earl has collected none of the acorns he needs. To what point does this correspond? Plot this point on your coordinate system.

c) After two days ($t = 2$), Earl has collected 8 acorns. Beginning at the previous point, move 2 days to the right and 8 acorns up. Plot this point. What are its coordinates?

d) Each time you go 2 days to the right, you must move 8 acorns up and plot a point. Continue doing this until you reach 14 days.

e) Draw a line through your data points.

f) Use the graph to determine how many acorns he will have collected after 10days. Will Earl have collected enough acorns for his winter hibernation?

After 10 days he will…
Yes. He will have collected enough…. No. He won't have collected enough…

g) The number of acorns collected is increasing at a rate of 8 acorns every 2 days. Reduce this to a rate that shows the average number of acorns that is collected each day.

h) The table below lists the number of acorns Earl will have collected at various times. Some of the entries have been completed for you. For example, at $t = 0$, Earl has no acorns, so $N = 0$. After one day, the amount increases by 4, so $N = 0 + 4(1)$. After two days, two increases have occurred, so $N = 0+4(2)$. The pattern continues. Fill in the missing entries.

t	N
0	0
1	$0 + 4(1)$
2	$0 + 4(2)$
3	$0 + 4(3)$
4	
6	
8	
10	
12	
14	

i) Express the number of acorns collected, N, as a function of the time t, in days.

j) Use your function to predict the number of acorns that Earl will have after 10 days. Does this answer agree with your estimate from part (f)?

Yes, this answer agrees with the previous one.

No, this answer doesn't agree with the previous one.

- ACTIVIY6. On June 16, 2006, the conversion rate from Euro to U.S. dollars was approximately 0.8 to 1: every0.8 Euros were worth 1 U.S. dollar. (1 $=0,8 €).

1 dollar=0,8 euros

a) Choosing dollars to be the independent variable and Euros to be the dependent variable make a graph of coordinate system. Mark every dollar on the dollar axis and every 0.8 Euros on the Euro axis.

b) Zero dollars are worth 0 Euros. This corresponds to the point (0, 0). Plot it on your coordinate system.

c) One dollar is worth 0.8 Euros. Plot this as a point on your coordinate system.

d) For every dollar you move to the right, you must go up 0.8 Euros and plot a point. Do this until you reach $10.

e) Draw a line through your data points.

f) Use the graph to estimate how many Euros $8 are worth.

g) Use the graph to estimate how many dollars 5 Euros are worth.

h) The following table shows some values of dollars and their corresponding value in Euros. Fill in the missing entries

Dollars	Euros
0	0
1	$0 + 0.8(1)$
2	$0 + 0.8(2)$
3	
4	
5	
10	

i) Use the table to make an equation to convert dollars to Euros.

j) Use the equation from (i) to convert $8 to Euros. Does your answer agree with the answer from (f) that you obtained using the graph?
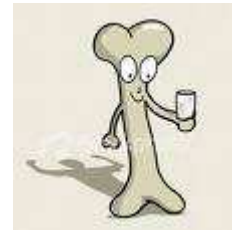
Yes. It agrees with….
No. It doesn't agree with…

- ACTIVIY 7. Work in pairs and later we will report the results in the plenary.

Anatomy. Anthropologists use the length of certain bones of a human skeleton to estimate the height of the living person.

One of these bones is the femur, which extends from the hip to the knee.

To estimate the height in centimetres of a female with a femur of length "x", this function can be used:

h(x)=61,41 +2,32x
h= woman's height in cm
x= femur's length in cm

a) Draw the graph.

b) What's the y-intercept?

c) What's the x-intercept?

d) What's the gradient?

e) What was the height of a female whose femur measures 46 centimetres?

The woman was about……….centimetres tall.

- ACTIVIY 8. Work in pairs. Then we will report the result in the plenary.

Safety. The time in seconds that a traffic light remains yellow is given by the function:

$$t(s) = 0{,}05\,s + 1$$

s = the speed limit in miles /hour

a) Draw the function

b) Indicate the y-intercept, x-intercept, and gradient.

c) How long will a light remain yellow if the speed limit is 45 miles per hour?

The light will remain yellow…

- ACTIVIY 9. Work in pairs, later we will report the results in the plenary.

Health. A person's normal systolic blood pressure "p" depends on the person's age "a". To determine the normal systolic blood pressure for an individual, you can use the equation:

$$p = 0{,}5\,a + 110.$$

a) Write this relation in functional notation.

b) Find the normal systolic blood pressure for a person who is 36 years old.

c) Graph the relation. Describe the graph.

- ACTIVIY 10. Work in pairs and then we will report the results in the plenary.

Travel. The cost of a one-day car rental from City-Wide Rentals is given by the function:

C(x)= 0,18x +35

x=number of km that the car is driven

C= cost in Euros

Suppose Ms. Burton drove a distance of 95 km and back in one day.

a) What is the cost of the car rental?

b) Draw the function, and indicate the x-intercept, y-intercept, and gradient.

- ACTIVIY 11. Work in pairs and then we will report the results in the plenary.

Animals. The fastest swimming bird is "the gentoo penguin" of Antarctica.

It moves through the water at about 27,36 kilometres per hour.

a) Calculate the constant of variation

b) Write the equation

c) Draw the graph

d) How far can the gentoo penguin swim in 45 minutes? Try to calculate it through the graph first, and then through the equation.

e) After reading the following texts, prepare questions about the text and answer them.

Who…?	Where…?	What…?
How much/many…?	How…?	Why?

ANCTARTICA. THE WORLD LAST GREAT WILDERNESS

Antarctica is a frozen, windswept continent, so hostile and remote that it has no permanent inhabitants.

Scientists working here have made many discoveries from studying Antarctica's land and atmosphere, and from clues buried beneath the ice.

These discoveries also reveal signs of changes in the future that could affect us all.

Although there are no trees on Antarctica more than 100 million birds nest and breed there.

Using dogs to pull sledges ended in 1994 when all dogs were removed from Antarctica as part of an international agreement to protect the Antarctic environment.

About 1,200 people spend the winter on Antarctica - about a third are scientists and the rest are support staff.

Tourists on most ships clean their boots carefully every time they get on or off so that they don't spread seeds or diseases around different areas of the Antarctic.

Antarctica is designated as a natural reserve devoted to peace and science.

Antarctica's vast ice sheet is actually on the move. Due to gravity and the ability of the ice to 'flow', it creeps towards the coasts at an average rate of ten metres per year.

http://www.discoveringantarctica.org.uk/

- **ACTIVIY 12.** Work in pairs and then we will report the results in the plenary.

Business. Bongo´s Clown service charges 50€ for a clown to perform at a birthday party plus an additional 2€ per guest.

A) Write an equation that represents this situation. Let x represent the number of guests and let y represent the cost of a clown's services.

B) Draw the function.

C) Serena's parents paid 68 euros for a clown to perform at Serena's party. How many people attended Serena's party?

- **ACTIVIY 13.** Work in pairs and then we will report the results in the plenary.

Cost=2500€

500€

80€ per week

Anne is saving money to buy a 2500 euros second-hand car. She started with 500 euros and has been saving 80 euros per week since then.

The equation:

$y=500+80x$

represents her total savings "y" after any number of weeks x.

a) Draw the graph.

B) Use the graph to estimate when Anne will have enough money.

c) Verify your estimate using the equation.

- ACTIVIY 14. Work in pairs and then we will report the results in the plenary. Astronomy.

The weight of an object on Venus varies directly to its weight on Earth. An object that weighs 80 kg on Earth weighs 72 kg on Venus.

a) Find the constant of variation, and write the equation.

b) Draw the graph.

c) How much would an object weigh on Venus if its weight on Earth is 90 Kg?

The object would…..	weight weighted weigh	…Kg

- **ACTIVIY 15.** Work in pairs and then we will report the results in the plenary.

Measurements. If there are 1000 metres in one km:

a) Find the constant of variation, and write the equation.

b) Draw the graph

c) How many metres are in 4, 5 km?

There is… There are… There will be… There would be… There…	…metres	in…km

- ACTIVIY 16. Work in pairs and then we will report the results in the plenary.

Health. The height "h" of an average person varies directly with their foot length "f". Suppose that the constant of variation is 7.

a) Write an equation that represents this situation.

B) Draw the graph.

C) Use the graph to find the height of a person whose foot length is 24,13 centimetres.

- ACTIVIY 17. Work in pairs and then we will report the results in the plenary.

Life science. A full-grown male seal weighs about 250 kg. This is 10 kg less than the weight of a full-grown male sea lion.

A) Write an equation that represents the weight of a full-grown male sea lion.

B) Draw the graph

C) How much does a full-grown male sea lion weigh?

- ACTIVIY 18. Work in pairs and then we will report the results in the plenary.

There are two common systems for measuring temperature, Celsius and Fahrenheit.

Water freezes at 0° Celsius (0°C) and 32° Fahrenheit (32° F); it boils at 100°C and 212° F.

(a) Assuming that the Celsius temperature (TC) and the Fahrenheit temperature (TF) are related by a linear equation, find the equation.

(b) What is the slope of the line relating *TF* and *TC,* if *TF* is plotted on the horizontal axis?

(c) At what temperature is the Fahrenheit reading equal to the Celsius reading?

(d) Normal body temperature is 98.6°F. What is it in °C?

- ACTIVIY 19 . STRAIGHT-LINE BATTLESHIPS.

First, with the help of the whole class, I am going to write down the equation of straight lines in cards.

You are going to work in pairs. Each pair has to draw four points in the coordinate axes.

I will choose one of the cards, and you have to decide (by calculating or plotting) if the line passes through any of your points. If so, the ship is sunk (Then I will read aloud the rest of the equations).

The winner, of course, is the last remaining student with at least one undamaged battleship.
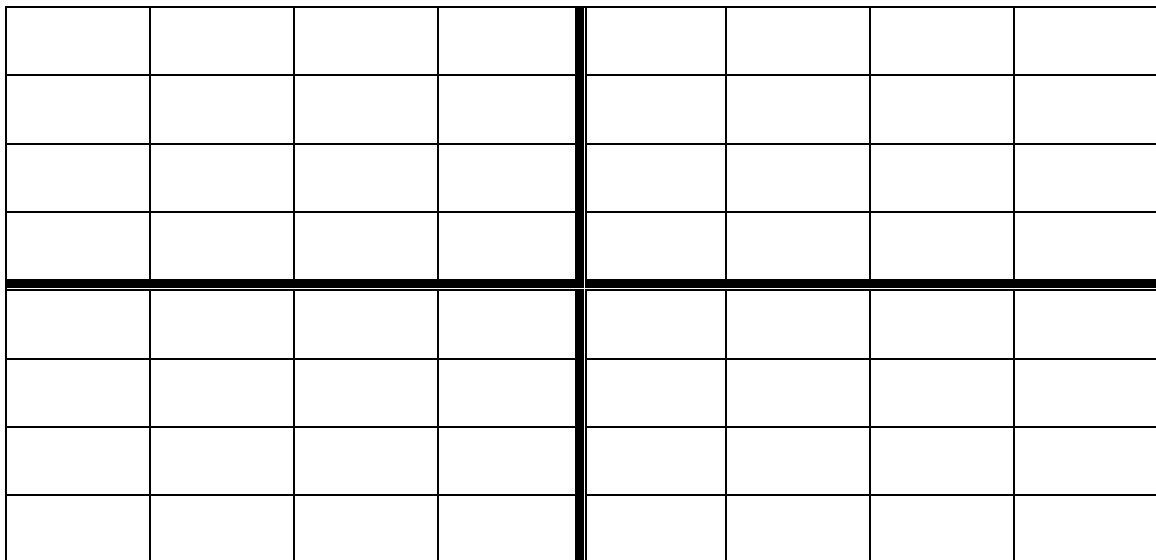
- **ACTIVIY 20.** We will divide the class in groups of six.

1. Write the following slopes on six individual pieces of paper:

$$m = \frac{2}{5} \qquad m = -\frac{2}{5} \qquad m = \frac{3}{4} \qquad m = -\frac{3}{4} \qquad m = \frac{1}{2} \qquad m = -\frac{1}{2}$$

Place each piece of paper face down and shuffle them. Each group member selects a slope. Do not show the slope to anyone in the group.

2. Similarly, on another six individual pieces of paper, write the following y-intercepts.

$$b = 2 \qquad b = -2 \qquad b = 1 \qquad b = -1 \qquad b = 4 \qquad b = -4$$

3. Write your Linear equation in the form of "y=mx + b", using the values of "m" and "b", that you had selected in the space provided below. Do not show this equation to your group.

4. Draw the graph of your equation neatly on the graph paper on the next page. Do not label the graph with the equation.

5. When everyone in the group has finished, place the graph in a pile. The group as a whole should identify the slope and y-intercept and the correct equation of the graph.

What's the…?
Where does/do the line cross/crosses…?
How many units change "y", if x increases…unit/s?
Do/Does the point…belong/s to the function?

Yes. I agree with you
No. I disagree with you/I don't agree with you.
I think that the correct answer is…
You are wrong. The right answer is…
You made a mistake in your answer because…
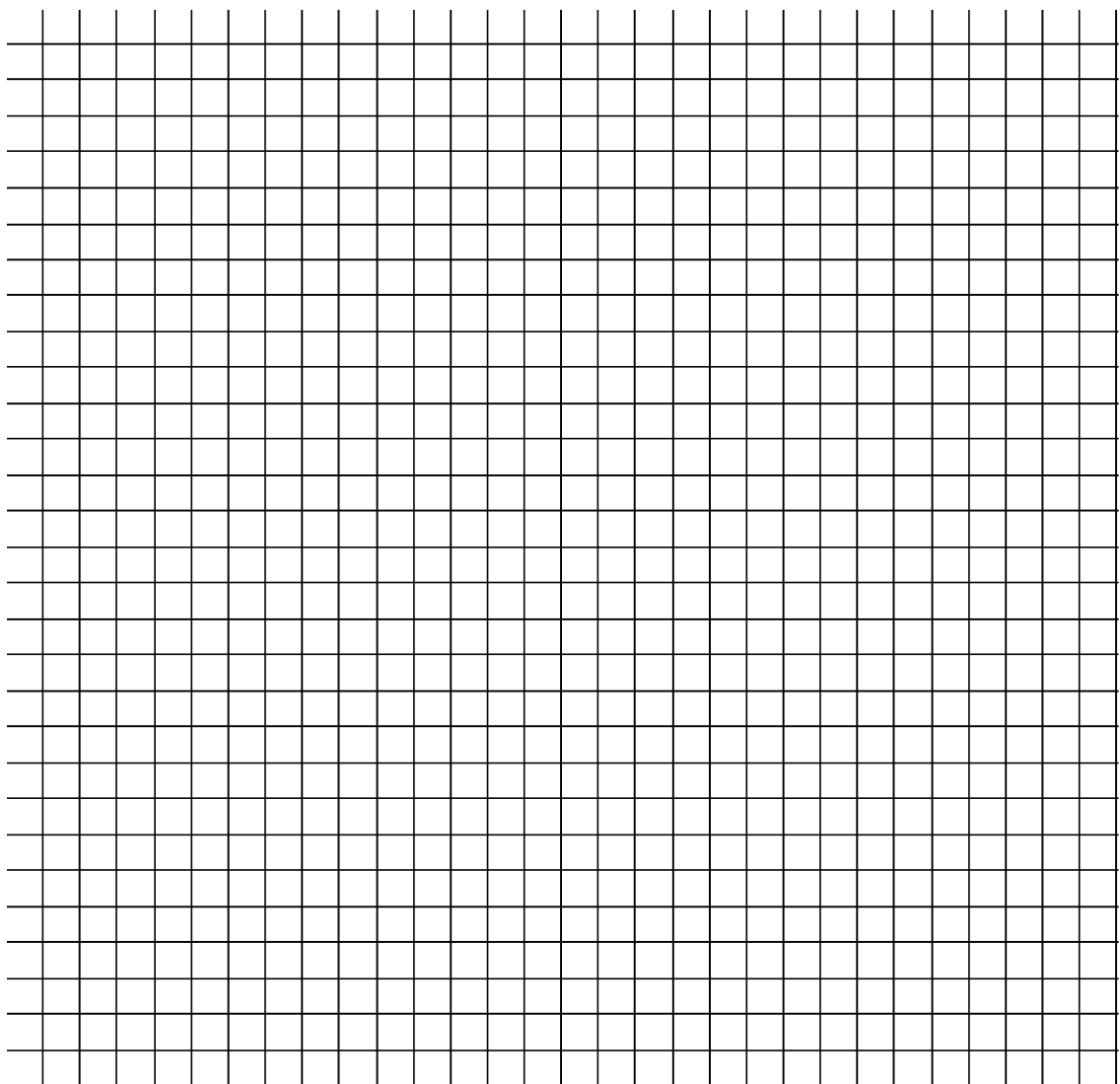
- ACTIVIY 21. I am going to give you 12 sets of cards, with a linear graph, a brief description of a real situation and a linear function. You are going to work in pairs (later we will report the results in the plenary), and you have to sort the cards into groups of three, and justify your choice (the cards are at the end of the lesson).

This graph/this sentence/this equation corresponds to….because…
This sentence means that…
This equation means that…
The dependent variable is…and it's the same as….
The slope is….and it means that….

- ACTIVIY 22. **Analyzing and Choosing a First Job (work in pairs).**

Do you have a job? If not, what will your first job be? What expenses will you have? How much money will you actually earn? How can you compare earnings between two jobs? Linear equations can help you answer all these questions. We are going to imagine that we are living in the United States.

Find Out by Graphing

- Find the hourly wage (the salary per hour) for two jobs that interest you.
- Make a graph that shows the income for each job. Show hours worked (0 to 10) on the horizontal axis and income on the vertical axis.
- Suppose you work eight hours. Explain how your graph shows the difference in income from the two jobs.

Internet Tip: You can get actual income data for different occupations at "Occupational Employment Statistics List of Occupations of the United States": http://stats.bls.gov/oes/1999/oessrci.htm

Find Out by modelling.

Suppose you earn $5.50/hour at a bakery. When you get your first pay check you discover that $1.15/hour is subtracted for taxes and benefits. You work "x" hours during a five-day week, and you spend $3.75 each day for lunch.

- Write an equation for your earnings for a week after taxes and expenses.

- How many hours must you work to earn $120 after taxes and expenses?

Find Out by Interviewing

Interview an adult about a job he or she had as a teenager. Ask about positive and negative aspects of the job, salary, and expenses. Write an equation that describes the person's earnings after expenses for a week.

What did you do when you were working as.....?
How much money….?
How many hours per week…?
Did you like….? Why?
What would you do if…?
What did you spend your money on when…?
Did you help your parents…?
How much tax did you pay?
Did you travel to…?

Finishing the Project

- Work with several classmates. Share what you have learned about jobs for a teenager. List positive and negative aspects about income and expenses for several jobs.. Write some sentences that explain what job you would like and why.

POVERTY

Poverty is not having enough money to have important things like food, water, shelter, or toilets. Many people in different countries live in poverty, especially in developing areas of Africa, Latin America and Asia.

There are different ways to measure poverty. The World Bank says that extreme poverty is when someone needs to live on less than US$ 1 a day. Moderate poverty is when people need to live on less than 2 such dollars a day.

In the developed world many people are seen as the working poor. They have a job, but do not earn enough money. They need to spend a lot of that money for living expenditures, so that at the end of the day, little of it is left.

http://simple.wikipedia.org/wiki/Main_Page

a) Do you think that the wages in Spain are the same as in Bolivia? Or Senegal?

b) Try to find minimum of average wages in other countries of the world. Convert it into Euros, and compare the results.

c) Look for information about the working conditions in poorer countries. Are they the same as in Spain?

d) What do you know about Children under 16 working? Look for information on the net.

Useful WebPages:

http://banderasnews.com/0711/edat-juststopit.htm

http://www.unicef.org/protection/index_childlabour.html

http://www.eurofound.europa.eu/eiro/2005/07/study/tn0507101s.htm

- ACTIVITY 23. FINAL ACTIVITY.

a) Choose the correct term or expression to complete each sentence.

1. A (function, linear equation) is an equation whose graph is a straight line.

2. Equations that are functions can be written in (functional notation, functional value).

3. A direct variation is a linear function that can be written in the form (xy=k, y=kx)

4. The (first, second) coordinate in an ordered pair is called the y-coordinate.

5. The graph of a linear function is a (horizontal/vertical/straight) line.

b) Join these heads with the correct tails:

"y" varies …	…a measure of its inclination or steepness
The graph of a linear function is …	…positive in an increasing linear function
The y-intercept is…	…linearly with x in a linear function.
The gradient or slope indicates …	…no slope in a constant function
The slope of the line is …	…the vertical axis
The slope is …	…a straight-line (but neither horizontal nor vertical)
The slope is …	…through the origin (0,0).
There is …	…how many units increase or decrease "y", if we increase "x" in 1 unit.
The constant function crosses …	…directly proportional to "x" in a direct variation function.
The graph of a direct variation passes …	…negative in a decreasing linear function
"y" is …	...where the line crosses the y- axis

b) We are going to divide the class in groups of three. Each group has to prepare the questions related to the chart above. For example:

- What does the slope indicate?

- Where does the line cross the y-axis?

- In turns, the first group asks a question to the next group. If the answer is right, the group gets 5 points. Then the second group asks another question to the following group, and so on. The winner is the group with the maximum score.

- If the question is not correct, 5 point will be subtracted from the total number of points.

- If the answer is not correct, 5 points will be subtracted from the total number of points.

What's.....?	What's the definition of...?
Can you define...?	Where...?
Can you describe...?	Is it true that...?

c) We are going to divide the class in groups of three. Each group has to prepare a power-point presentation, summarizing the main points of the lesson (concepts and definitions, giving examples...) and besides, you have to invent a real life problem related to linear functions or direct variation functions, and solve it. Your presentation will be recorded on the power point.
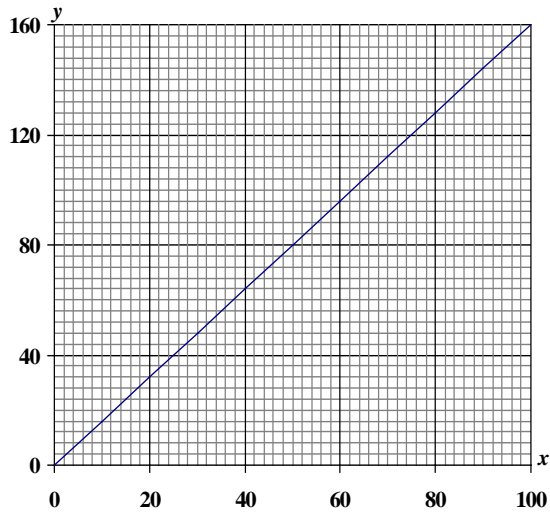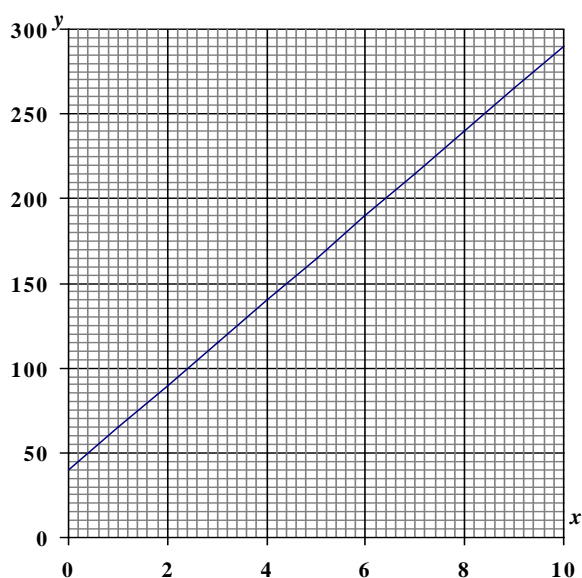
$y = 1.6x$

5 miles is equal to 8 km
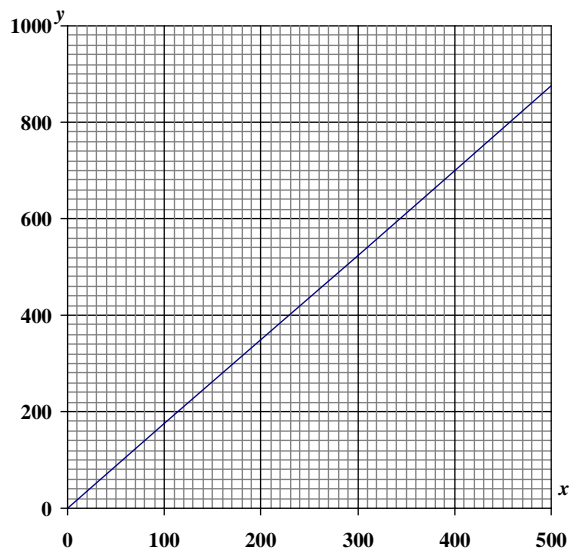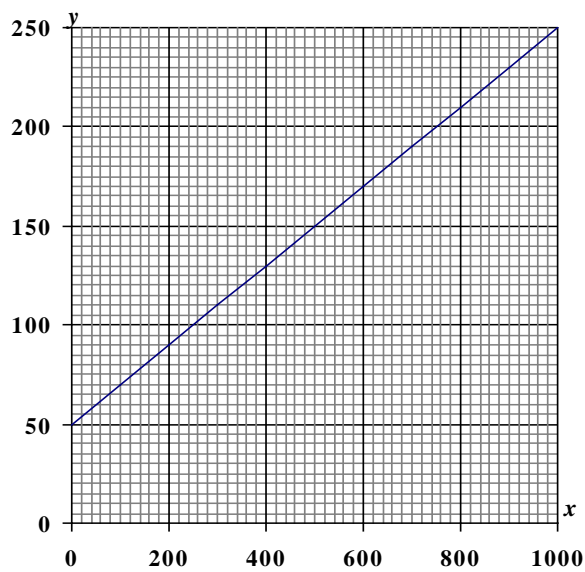

$y = 1.75x$

The Exchange rate is £1 = $1.75
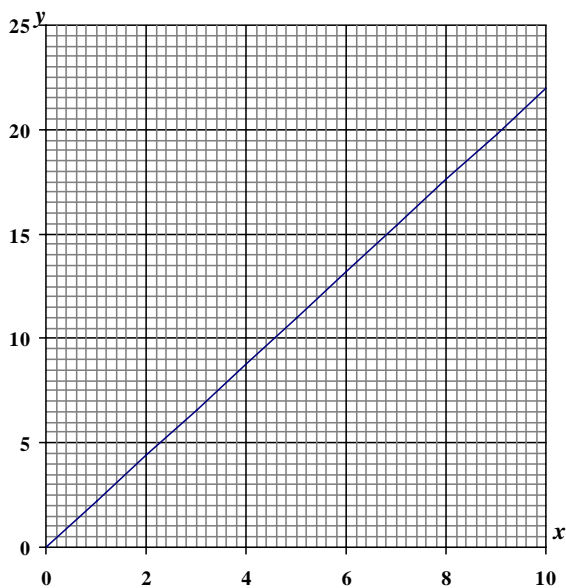
$y = 2.2x$

1 kilogram = 2.2 pounds
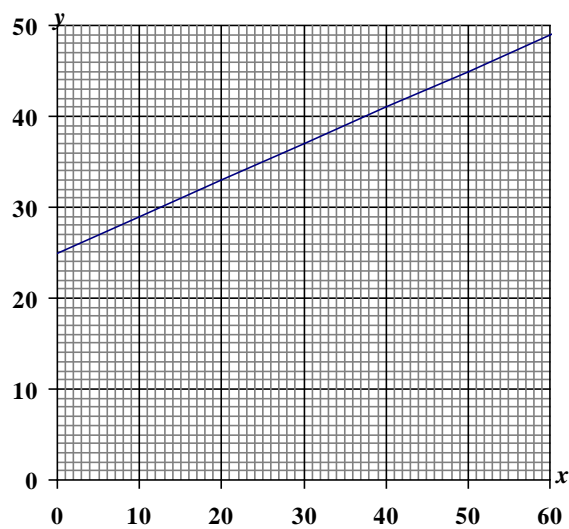

$y = 25x + 40$

The plumber charges £40 for a call-out plus £25 per hour.
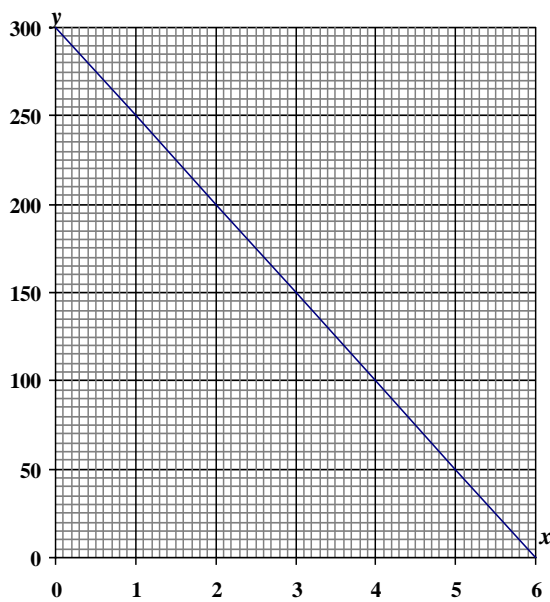
$$y = 0.2x + 50$$

The printing firm charges £50 for the design plus 20p per poster.
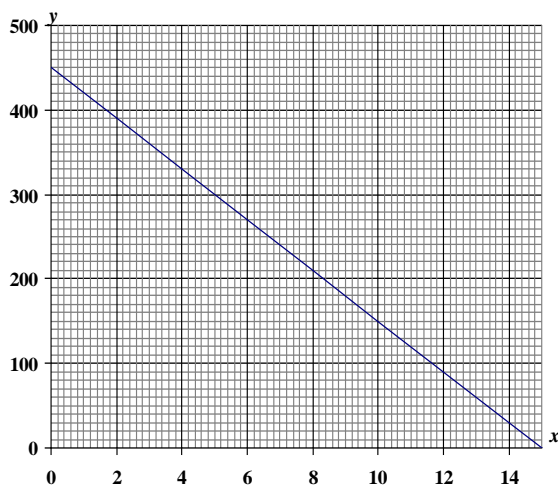

$$y = 0.4x + 25$$

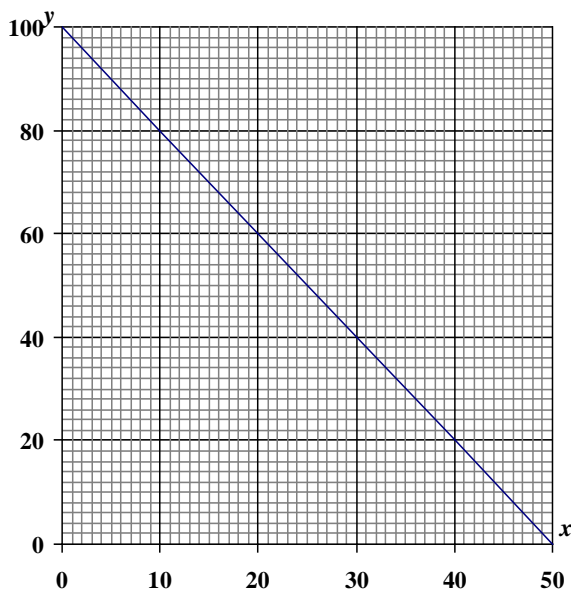**A delivery firm charges £25 plus 40p per mile.**

$$y = 300 - 50x$$

The journey is 300 miles long.
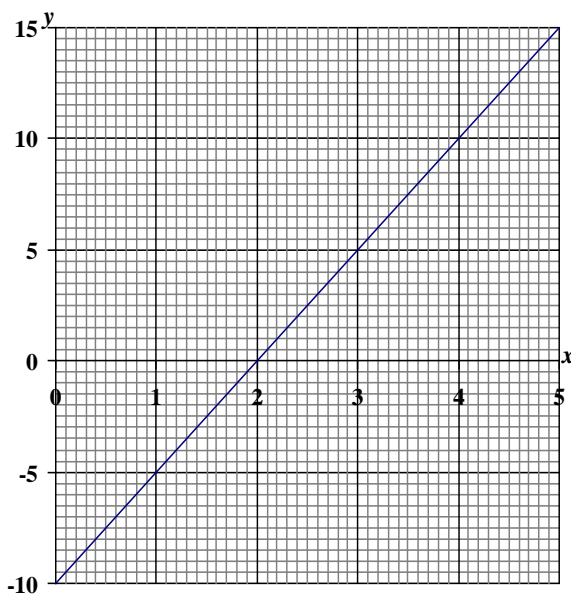We travel at 50 mph.

$$y = 450 - 30x$$

When full, the tank held 450 litres.
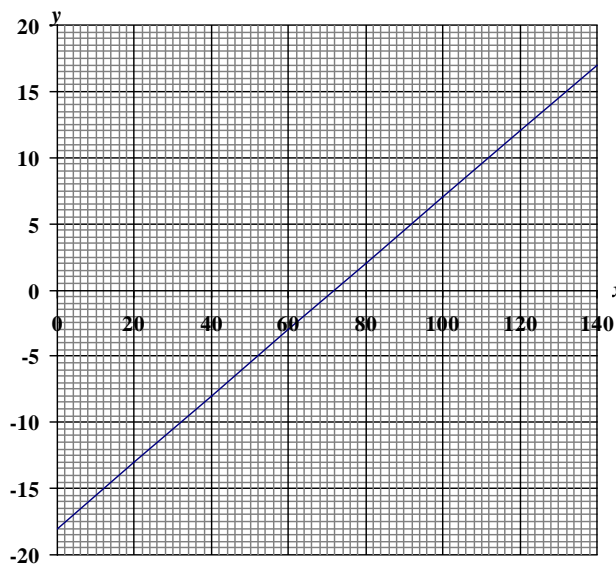We use 30 litres per day.

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$y = 100 - 2x$

The water boiled at 100°C.

It cools at a rate

of 2°C per minute

$y = 5x - 10$

It costs £10 to make the cheese.
We sell it at £5 per kilogram.
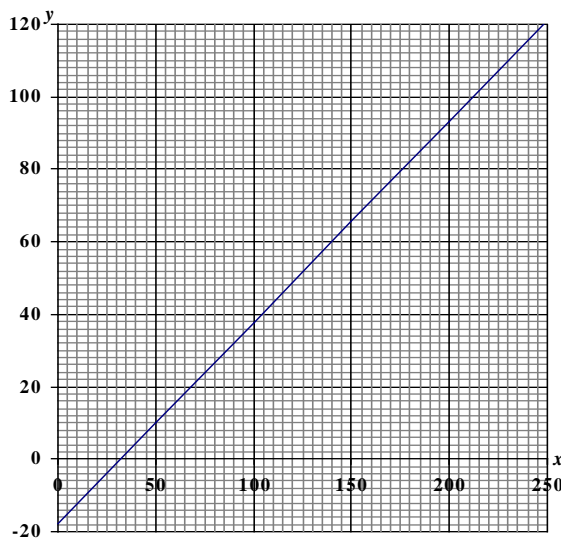If we sell more than 2 kilograms we make a profit.

$y = 0.25x - 18$

When we take it from the freezer its temperature is – 18ºC.

Its temperature rises at a rate

of ¼ºC per minute

$$y = \frac{5}{9}(x - 32)$$

To convert from Fahrenheit to Centigrade, subtract 32, then multiply by 5 and divide by 9.