STUDENT'S BOOK

Geometry and measurement

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Curs 2008-2009

Lesson 1

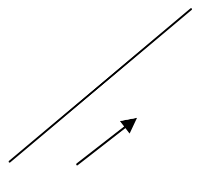
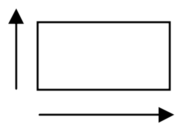
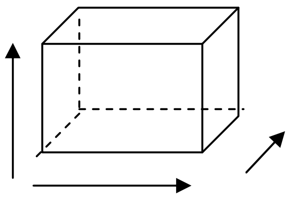
2D shapes: Introduction, classification, properties.

- **Before we start:** Dimension and look around
- **Worksheet 1**: Names and definition of a polygon
- **Worksheet 2**: Elements of a polygon
- **Worksheet 3**: Types of angles and lines
- **Worksheet 4**: Classifying polygons
- **Worksheet 5**: Triangles
- **Worksheet 6**: Angles and parallel lines
- **Worksheet 7**: More about triangles
- **Worksheet 8**: Pythagoras' Theorem
- **Worksheet 9**: Pythagoras' Theorem. Problems
- **Worksheet 10**: Quadrilaterals
- **Worksheet 11**: Polygons with more than 4 sides
- **Worksheet 12**: Circle and circumference

Activity 1. Match the shape with the dimension. The table below will help you.

- Rectangle
- Sphere
- Triangle
- Cube
- Segment

- One dimension
- 2D shape (Two dimensions)
- 3D shape (Three dimensions)

Zero dimensions	One dimension	Two dimensions 2D shapes	Three dimensions 3D shapes
			

Activity 2.

a. Choose objects in the classroom and classify them by their dimension, using the table below. *Try to be original and look for objects that your classmates find difficult to notice.*
b. Complete the table with the objects that your classmates have found.

One dimension	Two dimensions 2D shapes	Three dimensions 3D shapes

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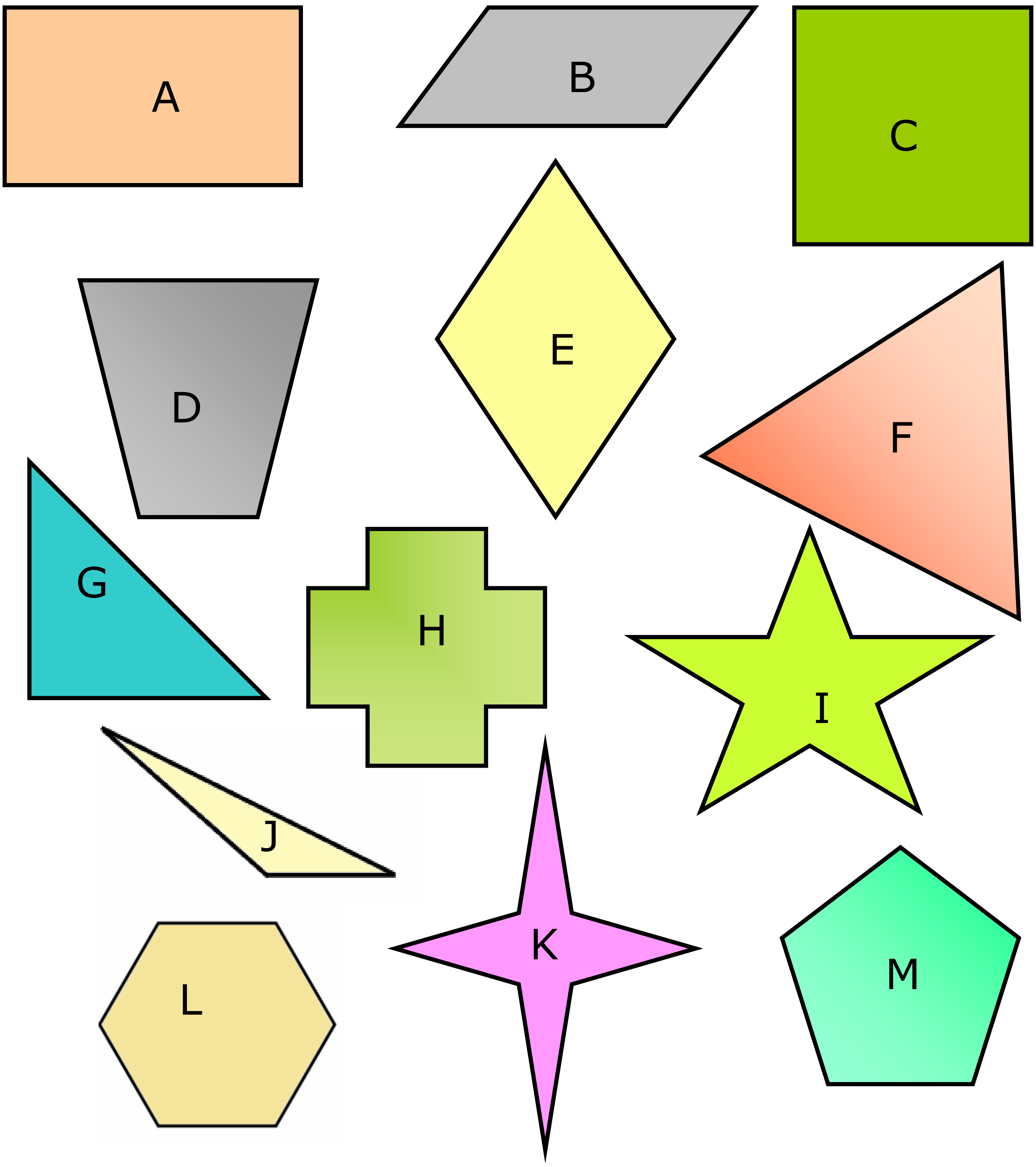
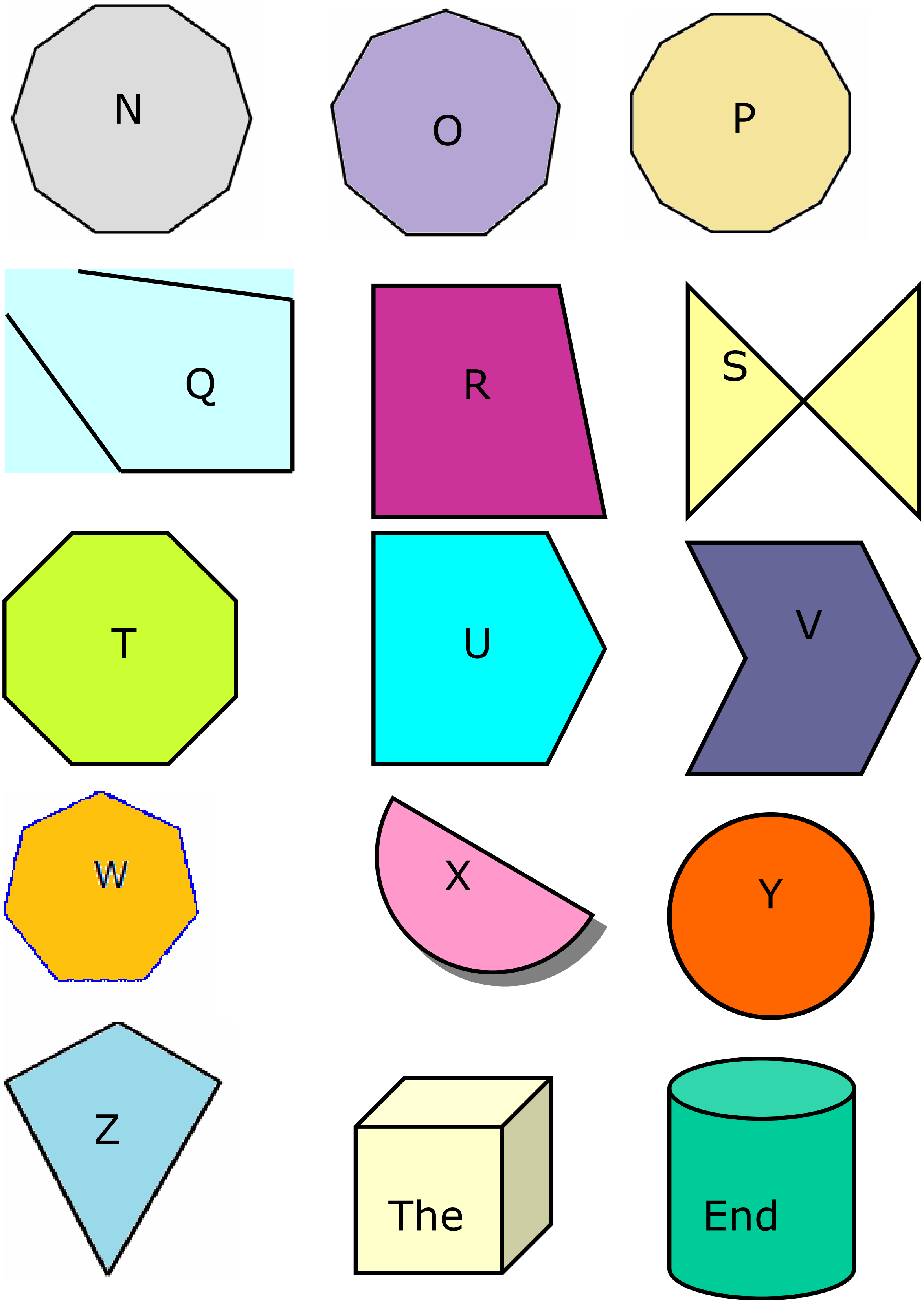
Look at all these shapes.
They will be our ***collection of shapes****.*




N
O
P
Q
R
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U
V
W
X
Y
Z
The
End

Activity 1 *Think and guess the names of our collection of 2D shapes*
Write the name inside each figure. Find the names on the list below.

SQUARE	CIRCLE	RECTANGLE
TRAPEZIUM (x2)	ROMBHUS	TRIANGLE (x3)
PENTAGON (x2)	HEXAGON (x2)	DODECAGON (x2)
OCTAGON (x2)	NONAGON	DECAGON (x2)
PARALLELOGRAM	SEMICIRCLE	CYLINDER
HEPTAGON	CUBE	??????????? (x2)
KITE		

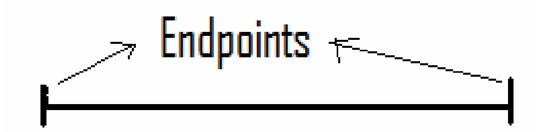
Here is some help! Some of these names have a Greek root:

- Penta = 5
- Hexa = 6
- Hepta = 7
- Octa = 8
- Nona = 9
- Deca = 10
- Hende = 11
- Dodeca = 12

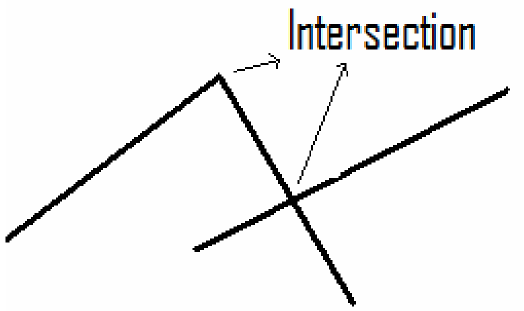
Activity 2
Read the following definition. Look at our collection of shapes and answer the questions.

A **polygon** is a closed 2D figure made by joining segments, where each segment intersects exactly two other segments.

A **segment** is a section of a line bounded by two endpoints.



The **intersection** of two lines is the point where they meet.

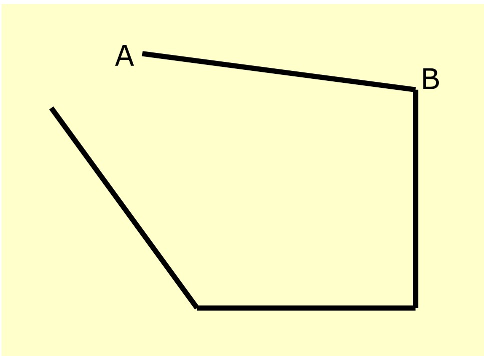


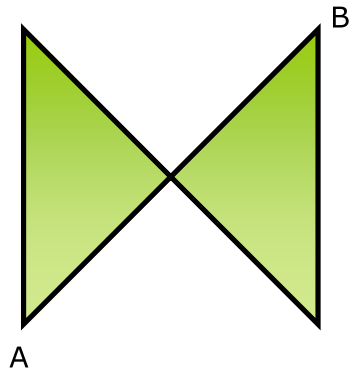
a. Are there any shapes that are not closed? Draw them:

b. Are there any 3D shapes? Draw and name them:

c. Are there any shapes made by curved lines? Draw and name them.

d. Segment AB, how many other segments intersect?





e. The six figures above **are not polygons**. Are the other ones polygons?

Complete the answer with:

- *are / are not,*
- *open / closed,*
- *2D / 3D,*
- *segments / curved lines,*
- *two / three*

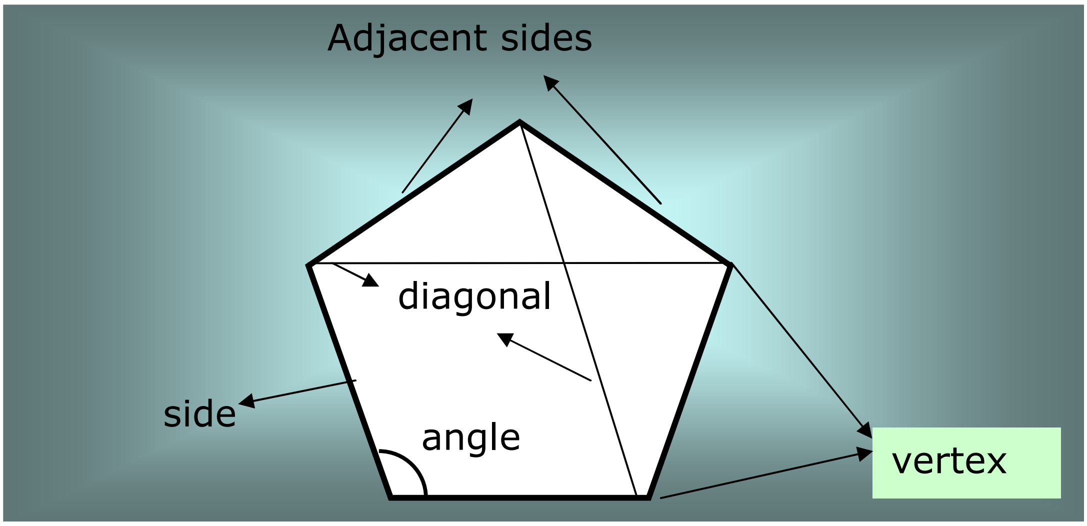
They ______________ polygons because they are ____________________

figures and there are __________ shapes, made with __________________

that intersect exactly ____________ others.

Activity1. *Elements of a polygon*

Look at this figure and fill in the gaps with the words that appear on it.



- A _________________ of a polygon is each one of the segments that form the polygon.
- Any two sides that share a common endpoint are called ___________________
- Two adjacent sides of a polygon meet in a point called _______________
- Two sides of a polygon form an ________________ of a polygon.
- A _________________ of a polygon is a segment connecting two non-adjacent vertices.

Activity 2

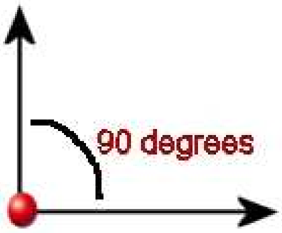
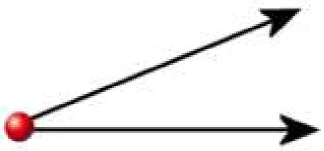
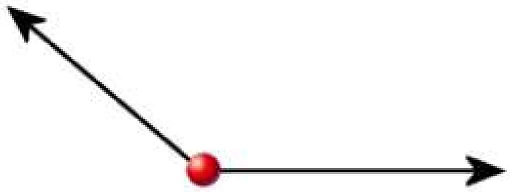
Fill in the grid and make sentences. The box helps you.

A	*square/ triangle* ……………	has	?	*Sides / vertices* …………

diagonals	pentagon	*A pentagon has **five** diagonals*
sides	parallelogram	
vertices	trapezium	
angles	triangle	
diagonals	square	

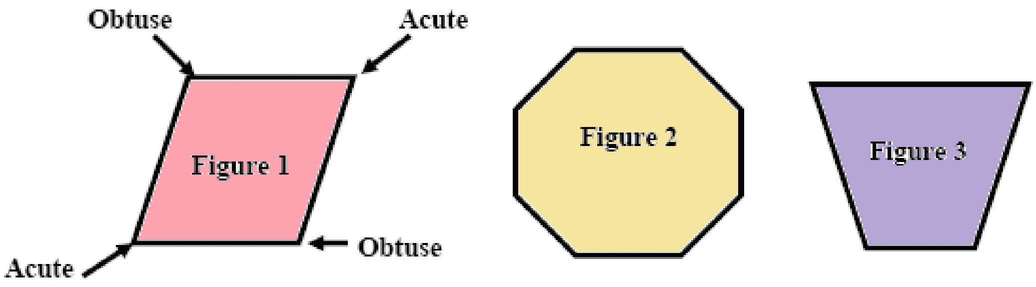
Activity 1

Look at these different types of angles and remember their names.

	
Right angle: measure 90 degrees	**Straight angle**: measure 180 degrees
	
Acute angles: measure less than 90 degrees	**Obtuse angle**: measure more than 90 degrees but less than 180 degrees

Activity 2

Look at the shapes below. Label each angle as in the example.



Activity 3

Draw 2D shapes in the table.

With four right angles	With one obtuse angle and two acute angles

Activity 4

There are many different types of lines: *vertical, horizontal, perpendicular, parallel*, etc… Draw and name the following:

a. Two lines that meet at a right angle.

They are ______________________________ lines

b. A line that runs from top to bottom (or up and down).

It is a ___________________________ line

c. A line that runs from side to side (or left to right)

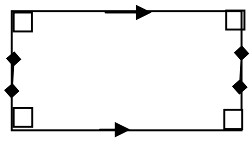
It is a ___________________________ line

d. Two lines that never will meet.

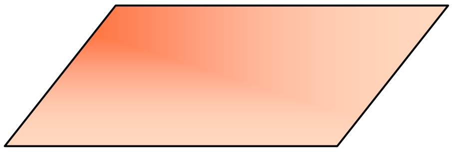
They are ______________________________ lines

Activity 5

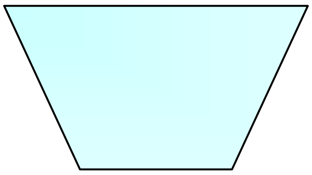
Look at the shapes below. Can you identify the shapes that have parallel lines? Can you identify the shapes that have perpendicular lines? Use the symbols as in the first shape.



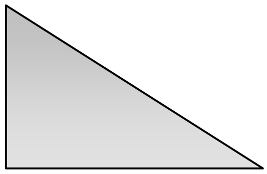
parallel / perpendicular / neither/ both



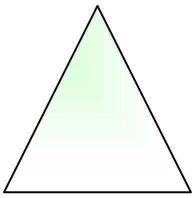
parallel / perpendicular / neither/ both



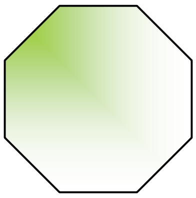
parallel / perpendicular / neither/ both



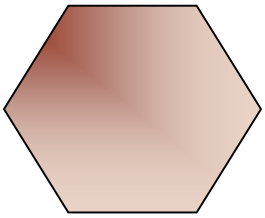
parallel / perpendicular / neither/ both



parallel / perpendicular / neither/ both



parallel / perpendicular / neither/ both



parallel / perpendicular / neither/ both

parallel / perpendicular / neither/ both

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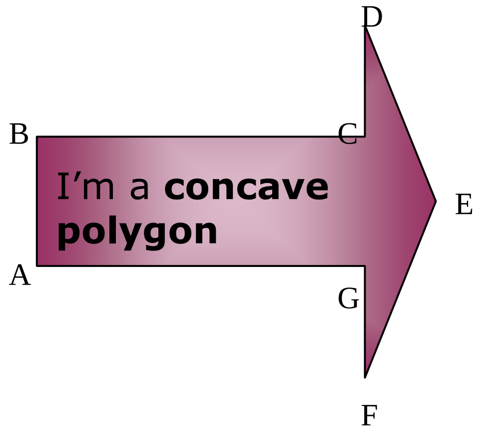
Activity 1. *Convex and concave polygons*

a. Write what kinds of angles there are on the figure below.



A is a ________________ angle

B is a ________________ angle

C is a ________________ angle

D is a ________________ angle

E is a ________________ angle

F is a ________________ angle

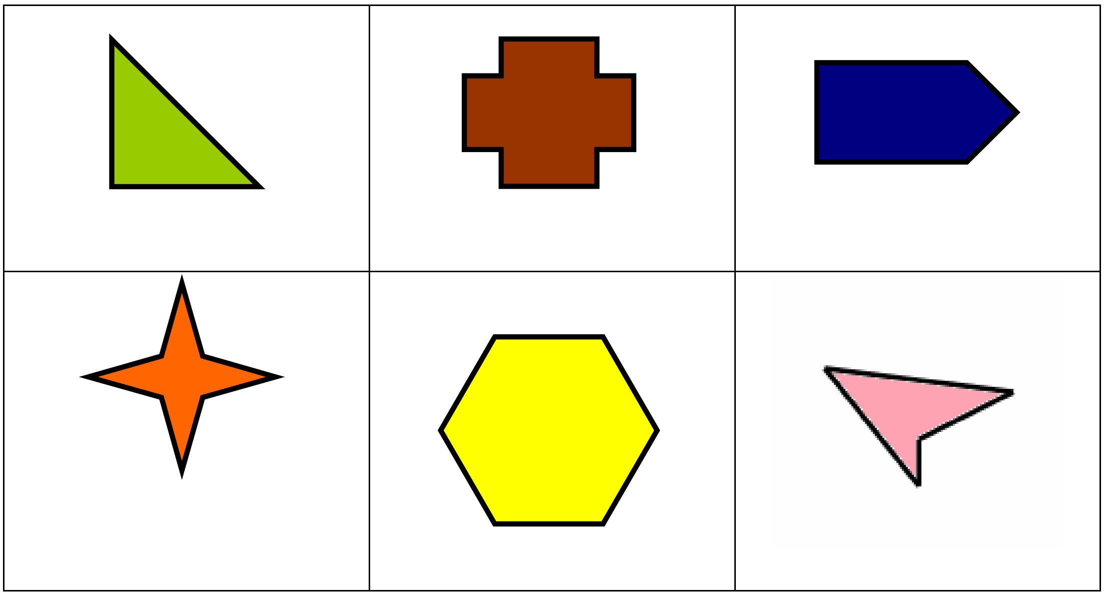
G is a ________________ angle

- A **reflex angle** is bigger than 180 degrees.

b. There are other reflex angles in the shapes of our collection. Draw the polygon V and describe its angles. This is another concave polygon.

- **Convex polygons** have no reflex angles
- **Concave polygons** have, at least, one reflex angle

c. Classify the following polygons into concave or convex.



Activity 2. *How many sides do they have?*

a. Another way of classifying polygons is according to how many sides they have. Fill in the following table as on the example.

Number of sides	Name	Polygon
3	Triangle	F, G, J
4	Quadrilaterals	
5		
6		
7		
8		
9		
10		
11	Hendecagon	________
12		

b. Talk in your group:

- About the differences you can find between the two pentagons.
- About the differences you can find between the two hexagons.

Activity 3. *Regular and irregular polygons*

a. Read the following definition.

A **regular polygon** is a polygon whose sides are all the same **length** and the angles are all equal.	 The **length** is • the distance from one end to another. • how long something is.

b. Measure the length of the sides of the polygons of our collection. Look at the angles. Think and fill in the table.

Regular polygons	C,
Irregular polygons	A,

c. Are the following regular or irregular polygons? Why?
Use the words in the definition or in the example to answer the questions.

- **The trapezium** is not a regular polygon because its angles are different and its sides are not the same length.
- Square
- Rectangle
- Rhombus

d. Come back to activity 2b and now write down your answer.
The difference between the two pentagons/hexagons is that one is

________________ and the other one is _______________________

Activity 4

Fill in the table with these figures: A, B, C, D, E, F, G, J, K, L, M, R, T, U, V and Z.

Number of sides	Regular		Irregular	
	Concave	Convex	Concave	Convex
3				
4				
5				
6				
8				

Activity 5

Draw a 2D shape that suits each case.

A concave quadrilateral	A regular triangle
A concave pentagon	

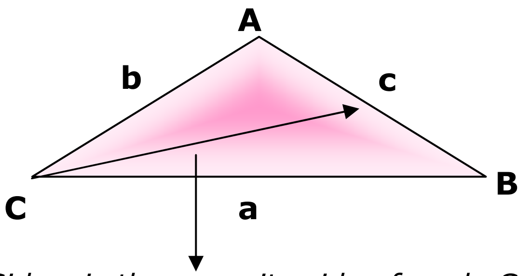
Activity 1

Discuss with your group and write down some notes on what you remember about triangles.

Activity 2

Read the following information and draw the EFG triangle; name its vertices and its sides.

A triangle is a polygon with three sides and three angles.



Side c is the opposite side of angle C

In a triangle:

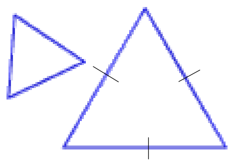
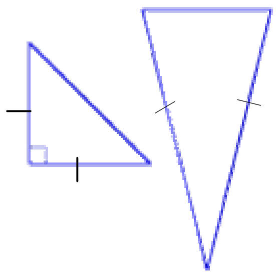
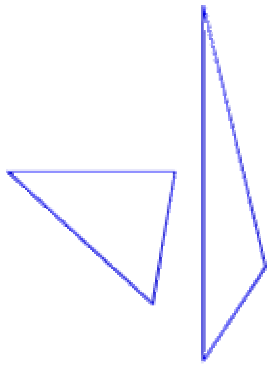
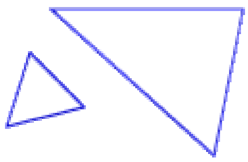
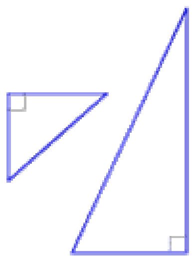
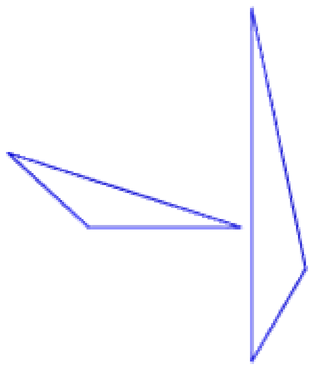
We use capital letters for the vertices and we name the opposite side of each vertex with the same letter in small letters.

The triangle on the left is the ABC triangle.

Activity3

Look at the table and remember what you know about classifying triangles. Classify the triangles of our collection of shapes filling the blanks in the sentences below.

There are three special names given to triangles that tell how many sides (or angles) are equal. Triangles can also have names that tell you what type of angles are inside.

	Types of triangles		
BY THEIR SIDES	**Equilateral triangle:**  A triangle having all three sides of the same length.	**Isosceles triangle:**  A triangle having two sides of the same length.	**Scalene triangle:**  A triangle having all three sides of different length.
BY THEIR ANGLES	**Acute triangle:**  A triangle having all acute angles.	**Right triangle:**  A triangle having a right angle	**Obtuse triangle:**  A triangle having an obtuse angle.

(The segments crossing the sides show equal sides. The square in an angle means a right angle.)

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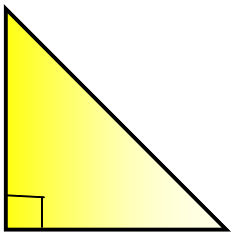
- The triangle F is a ________________ triangle because it has ________ ________________________________ and is a ______________ triangle because it has ____.
- The triangle G is a ________________ triangle because it has ________ ______________________________ and is a ________________________ triangle because it has ____.
- The triangle J is a ________________ triangle because it has ________ ____________________________________ and is a __________________ triangle because it has ____.

Activity 4

Sometimes a triangle has two names, for example:



Right Isosceles Triangle

Has a right angle and two equal sides.

Define the following combined names of a triangle as in the example. Draw the triangles.

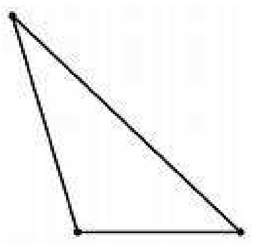
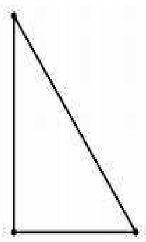
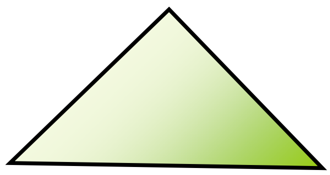
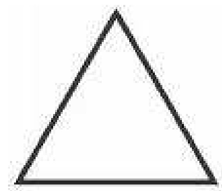
- Obtuse Scalene Triangle:

- Acute Isosceles Triangle:

- Right Scalene Triangle:

Activity 4

Classify the triangles. Use a ruler and a protractor.

A	B	C	D
			

A:

B:

C:

D:

Activity 5

Draw two triangles. Swap your worksheet with your partner and classify their triangles. Swap again and discuss the answer with your partner.

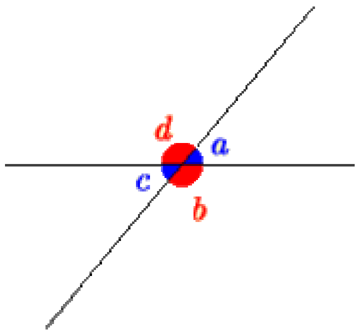
Activity1

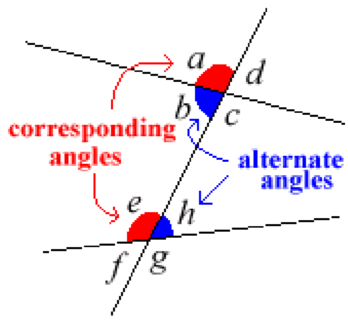
Read the following definitions and complete the two last ones with a sketch showing each situation.

Vertical Angles, Transversal lines, Corresponding and Alternate angles

<table>
<tr><td>When two straight lines cross each other they form 4 angles. Angle a and angle c are a pair of vertical angles.
So are angle d and angle b.

Vertical angles are equal in size.

$c=a$; $d=b$</td><td></td></tr>
<tr><td>A line passing through two or more lines is called a transversal.
When a transversal cuts two lines they form 8 angles:

Corresponding Angles
a and e are corresponding angles; d and h, b and f, c and g are also corresponding angles.
Alternate Angles
b and h are alternate angles; c and e are also alternate angles.</td><td></td></tr>
<tr><td>Corresponding Angles and Parallel Lines
When a transversal cuts two parallel lines, corresponding angles are equal in size.

Alternate Angles and Parallel Lines
When a transversal cuts two parallel lines, alternate angles are equal in size.</td><td></td></tr>
</table>

Activity 2

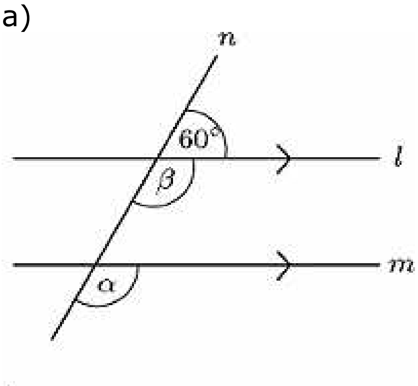
Draw a sketch showing each situation

a) Angles at a point add up to 360°	b) Angles on a straight line add up to 180°

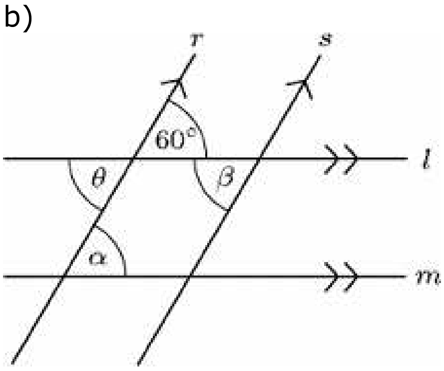
Activity 3

Find the missing angles in each of the diagrams below.

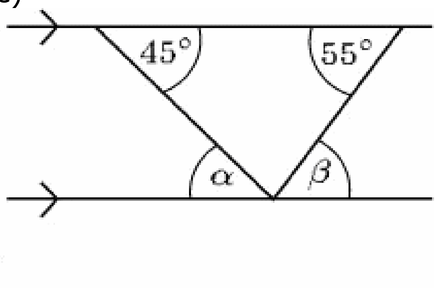
a)



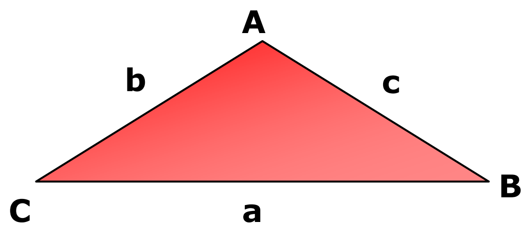
b)



c)



Look at these three properties of triangles

In a triangle: 1. There is a connection between the length of the sides and the size of the angles. The side opposite to the bigger angle is longer, the same with the small one. As **A** is the bigger angle, **a** is the longest side. And also, as **B = C** then **b = c** 2. The sum of two sides is always bigger than the remaining side. a + b > c b + c > a a + c > b 3. The three angles always add to 180°: **A + B + C = 180°**	

Activity 1

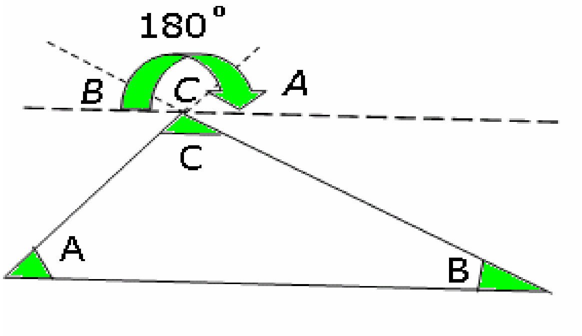
a) Draw a triangle whose sides measure 6 cm, 5 cm and 3 cm. Follow the steps.

1. Draw a segment AB with the same length as the longest side. Write the letters A and B on the endpoints of the segments
2. With the centre in A draw a circle of 5 cm of radius.
3. With the centre in B draw a circle of 3 cm of radius.
4. Label one of the points where the two circumferences meet with the letter C.
5. Join A and B with C.

b) Draw a triangle whose sides measure 6cm, 2 cm and 3 cm.

c) Why can you not draw it?

Activity 2
In the box below there is proof that **the angles in a triangle add to 180°**
Read it and use your own words to explain it to your classmates.



The top line (touching the top of the triangle) is running **parallel** to the base of the triangle.
So :

- angles A are the same because they are corresponding angles.
- angles B are the same because they are corresponding angles.
- angles C are the same because they are vertically opposite angles.

And you can easily see that A + C + B is a **complete rotation** from one side of the straight line to the other, or **180°**.

Activity 3
We can use that fact to **find a missing angle in a triangle**.

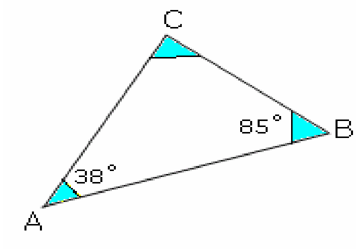
Example: Find the Missing Angle "C"

Start with: $A + B + C = 180°$

Fill in what we know: $38° + 85° + C = 180°$

Rearrange: $C = 180° - 38° - 85°$

Calculate: $C = 57°$

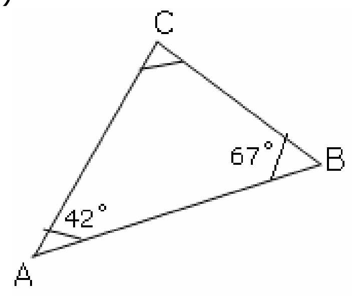


Can you think of another way to find the missing angle? Show it here:

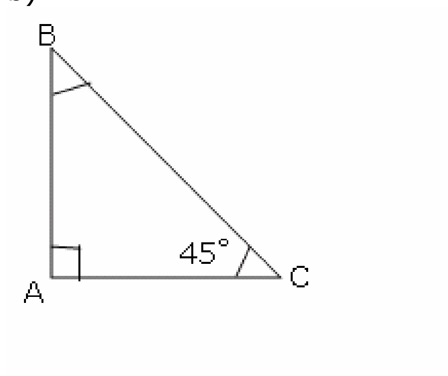
Activity 4

Find the missing angles on the triangles below.

a)



b)



Activity 5

Use the properties to answer the following questions in your group.

a) Find the size of the angles in an equilateral triangle.

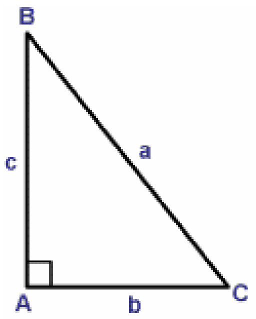
b) Which is the longest side in a right-angled triangle?

d) Find the size of the angles in an isosceles right-angled triangle.

e) Does a triangle with two obtuse angles exist? Why?

Pythagoras' Theorem

In a right angled triangle the square of the hypotenuse is equal to the sum of the squares of the other two sides.



So, the square of b (b^2) plus the square of c (c^2) is equal to the square of a (a^2):

$$\mathbf{a^2 = b^2 + c^2}$$

A is the right angle

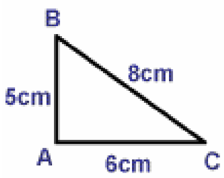
We can use the Pythagoras’ Theorem:

- To test whether a triangle has a right angle or not, because the theorem only works for right-angled triangles.
- To find the hypotenuse (the long side) of a right-angled triangle
- To find the short sides of a right-angled triangle.

Activity 1. Testing if the triangle is right-angled

Are the following triangles right-angled triangles? Look at the example and work out if exercise b and c are right-angled triangles.

a)



If a triangle is a right-angled triangle the Pythagoras’ Theorem must work. We have to check if it does with this particular triangle.

$a^2 = b^2 + c^2$

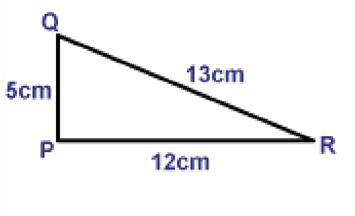
$a^2 = 8^2 = 64$

$b^2 + c^2 = 5^2 + 6^2 = 25 + 36 = 61$

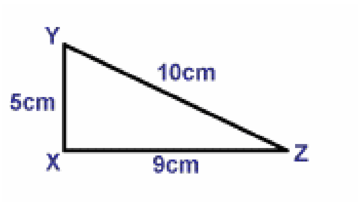
8^2 is not equal at $5^2 + 6^2$

so ABC is not a right-angled triangle.

b)

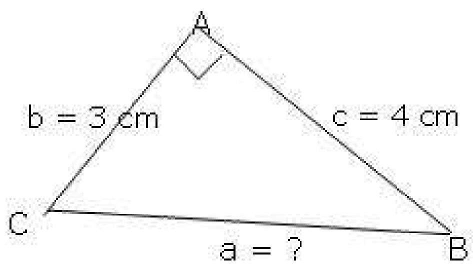


c)



Activity 2. Finding the hypotenuse

Look at the example and find the hypotenuse on triangles a and b.



1st step: Write down the formula

$$a^2 = b^2 + c^2$$

2nd step: Substitute

$$a^2 = 3^2 + 4^2$$

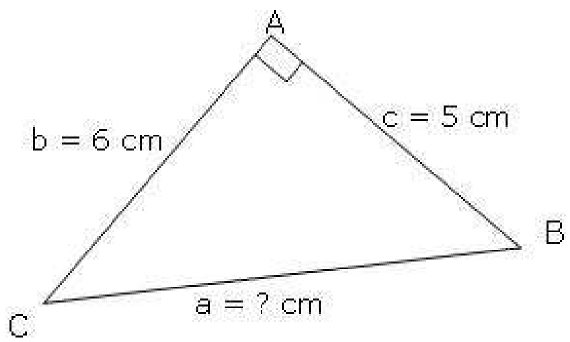
3rd step: Calculate

$$a^2 = 9 + 16$$

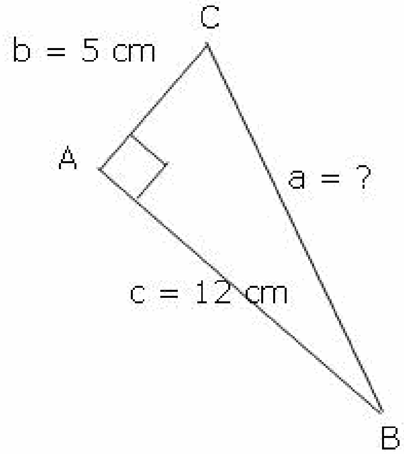
$$a^2 = 25$$

$$a = 5 \text{ cm}$$

a)

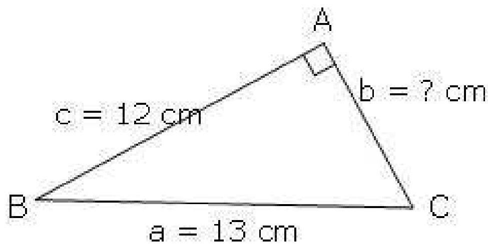


b)



Activity 3. Finding the short sides

Look at the example and find the short side missing on triangles b and c.



1st step: Write down the formula

$$a^2 = b^2 + c^2$$

2nd step: Substitute

$$13^2 = b^2 + 12^2$$

3rd step: Calculate

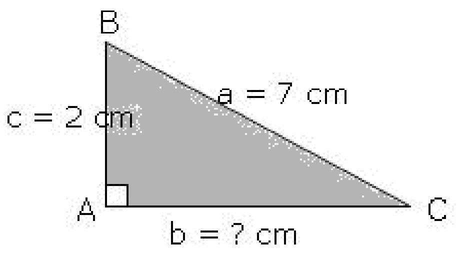
$$169 = b^2 + 144$$

$$b^2 = 169 - 144$$

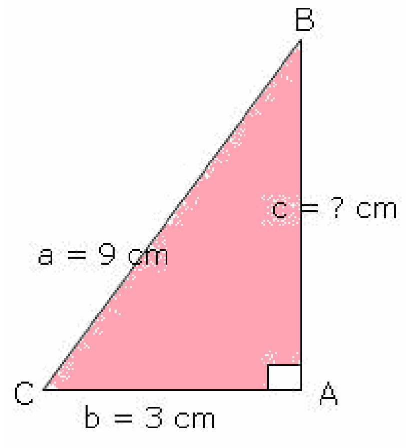
$$b^2 = 25$$

$$b = 5$$

a)



b)



Activity 4

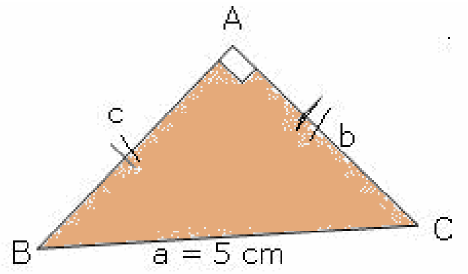
Solve these exercises. Start drawing the triangle, labelling its sides and angles as shown in activities 2 and 3, and then follow the steps.

a. In a right-angled triangle the length of the two short sides is 6 cm and 9 cm, find the length of the hypotenuse.

b. In a right-angled triangle the hypotenuse length is 23 cm, find one of the short sides, knowing that the other one is 15 cm long.

c. How long is the diagonal of a square whose side is 7 cm length?

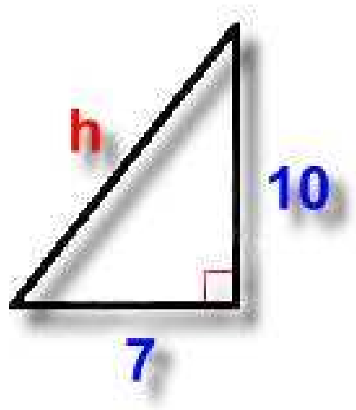
d. Find the missing side of this isosceles right-angled triangle.



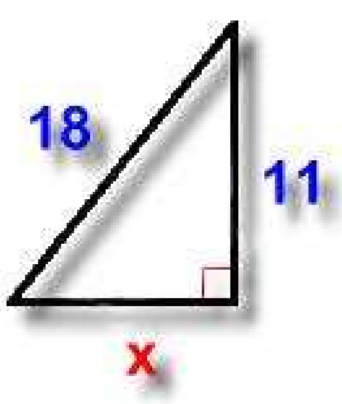
Activity 5. Revision

Find the missing sides in these triangles.

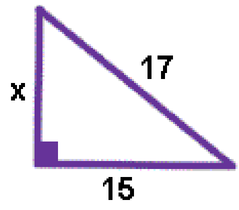
a)



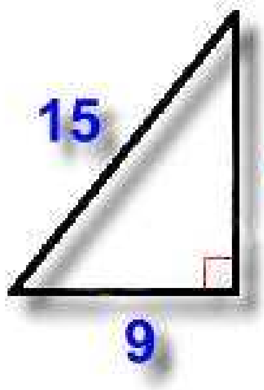
b)



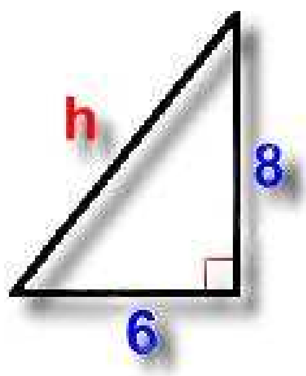
c)



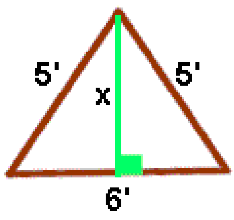
d)



e)

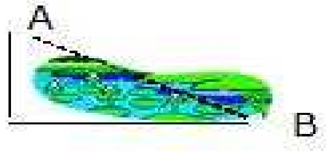


f)

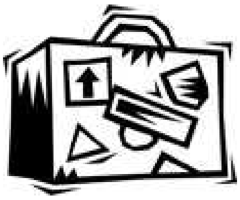


Pythagoras' Theorem Problems

1. To get from point A to point B you must avoid walking through a pond. To avoid the pond, you must walk 34 meters south and 41 meters east. How many meters would be saved if it were possible to walk across the pond?



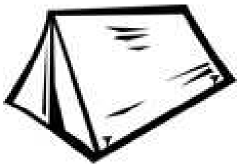
2. A suitcase measures 24 inches long and 18 inches high. What is the diagonal length of the suitcase?

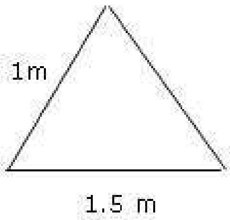


3. In a computer catalogue, a computer monitor is listed as being 19 inches. This distance is the diagonal distance across the screen. If the screen measures 10 inches in height, what is the actual width of the screen?

4. Two joggers run 8 miles north and then 5 miles west. What is the shortest distance they must travel to return to their starting point?

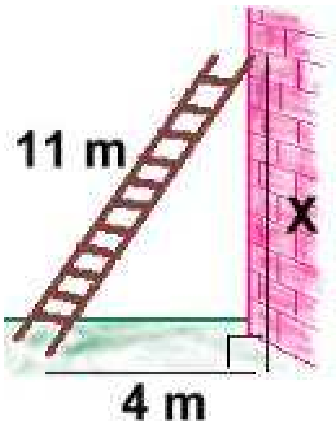
5. Oscar's dog house is shaped like a tent. The slanted sides are both 1 meter long and the bottom of the house is 1.5 meters across. What is the height of his dog house at its tallest point?





6. John made a small rectangular table for his workroom. The sides of the table are 120 cm and 50 cm. If the diagonal of the table measures 130 cm, does the table have right angles in its corners?

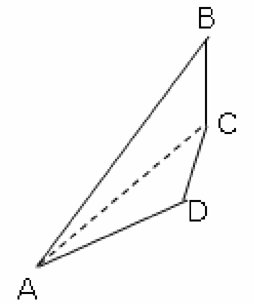
7. Look at the following picture and make up a problem to fit it. Then solve the problem.

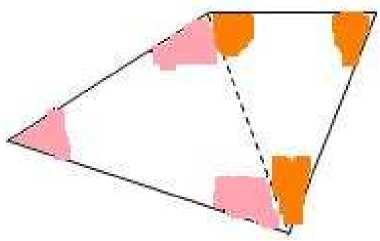


As you already know, shapes that have 4 sides are called **Quadrilaterals**
The quadrilaterals you are expected to know about are the **square**, the **rhombus**, the **rectangle**, the **parallelogram**, the **trapezium** and the **kite**.
Let us look at their properties.

Activity 1

Look at the pictures and complete the following sentence.



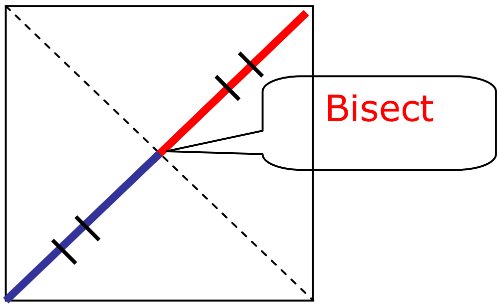


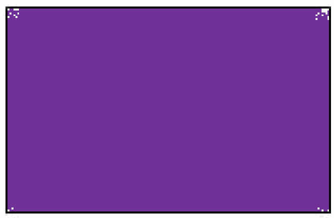
- The interior angles of a quadrilateral add up to _______ degrees because as we can see on the pictures it is formed by two triangles.

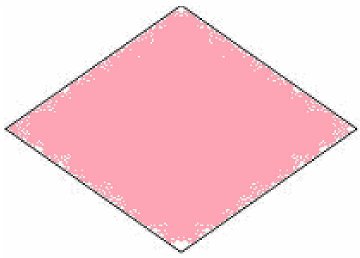
- If the angles of a quadrilateral are equal, each one measures _____

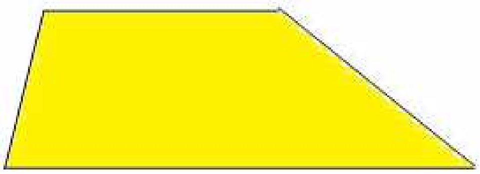
Activity 2

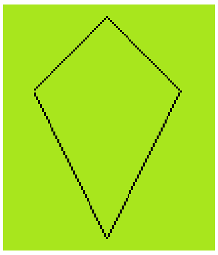
Draw the diagonals in the following quadrilaterals. Measure them and write down the measurements. In which quadrilateral do they bisect one another?







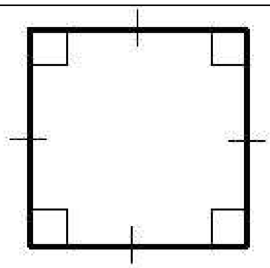
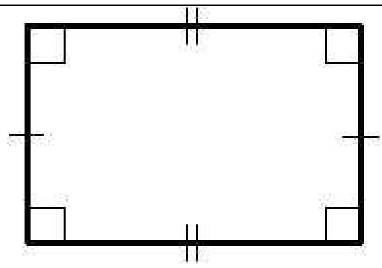
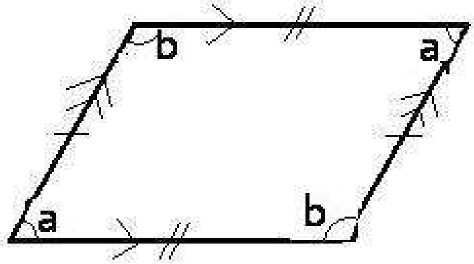
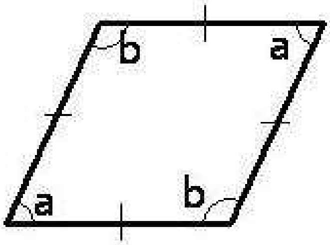
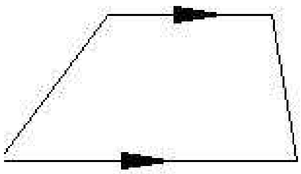
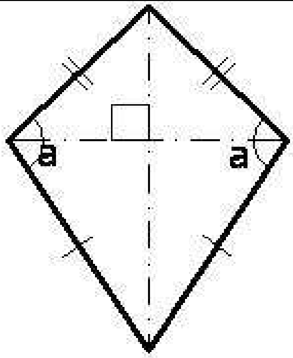




Ma Josep Sanz Espuny
IES Antoni Cumella (Granollers)

Activity 3

Look at the properties of each quadrilateral that are shown on the pictures and use the sentences in the box to describe the quadrilaterals.

		
		
□ *means "right angle"*	*The arrows mean parallel sides*	| and || *show equal sides*

All sides are of equal length. The diagonals bisect each other.

Opposite sides are parallel. The diagonals are equal in length.

The diagonals bisect each other at 90° . All sides are of equal length.

Only one diagonal is bisected by the other.

One pair of diagonally opposite angles is equal. Is a regular quadrilateral.

One pair of opposite sides is parallel. All angles are equal.

Diagonally opposite angles are equal. Opposite sides are of equal length.

Two pairs of sides are of equal length.

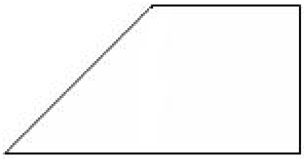
One pair of opposite sides is parallel. The diagonals cross at 90°.

• Square
• Rectangle
• Parallelogram
• Rhombus
• Trapezium
• Kite

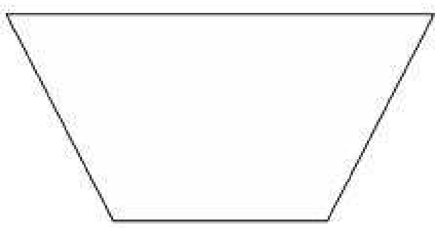
Activity 4

Describe the following trapezia.

a) Isosceles Trapezium



b) Right-angled Trapezium

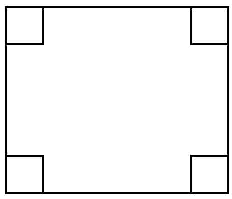


Activity 5. Happy enough?

Read and answer the questions

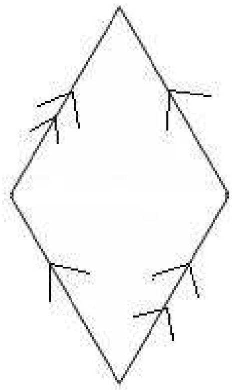
- **Square**

A square has 4 sides of equal length and 4 right angles.



Why do we not say that a square has 4 sides of equal length? Why discuss the angles?

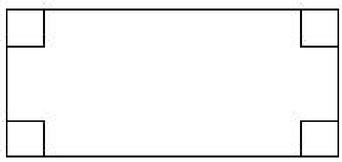
- **Rhombus**



A rhombus has got 4 sides of equal length and the opposite sides are parallel and opposite angles are equal.

What is the difference between a square and a rhombus?

- **Rectangle**

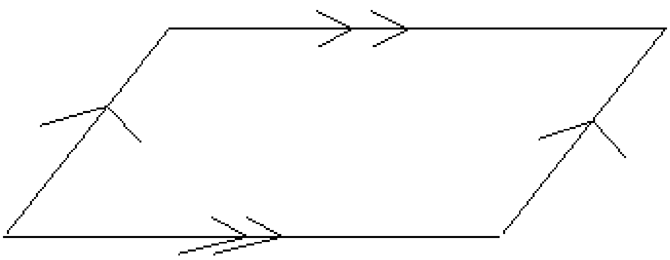


The rectangle has 2 pairs of equal sides, which are parallels and 4 right angles.

In a rectangle, what is the same as in a square and what is different?

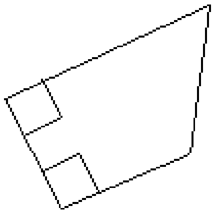
- **Parallelogram**

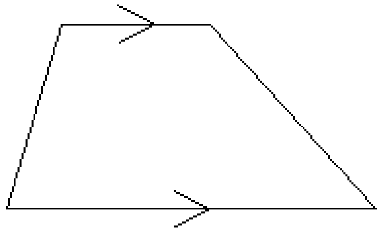
A parallelogram is a rectangle that has been pushed over. Opposite sides are the same length and they are parallel.



What have a parallelogram and a rhombus got in common?

- **Trapezium**



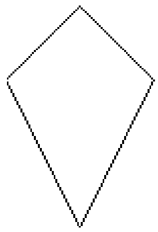


Both of these are types of trapezium. Each of them has different properties in the number of right angles. But each contains 4 sides and only two of them are parallel sides.

Which is the name of the first one? Is there any other special trapezium you know?

- **Kite**

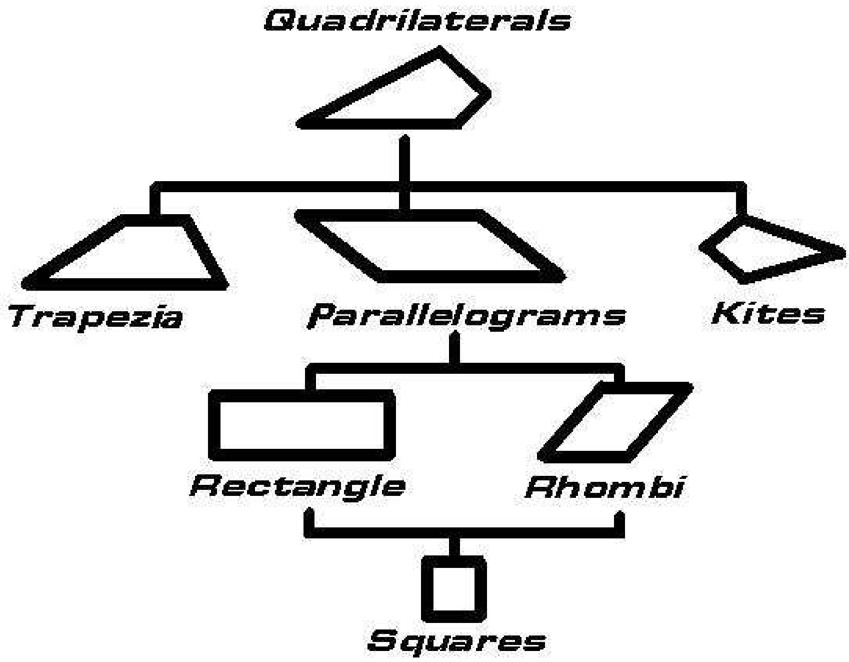
A kite has two pairs of adjacent sides that have equal length. But none of the sides are parallel.



Can you remember any property about its diagonals?

Activity 6

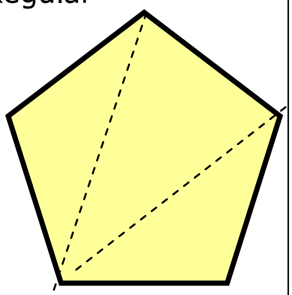
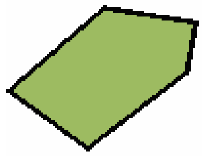
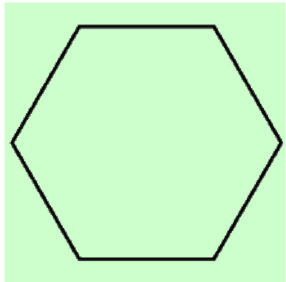
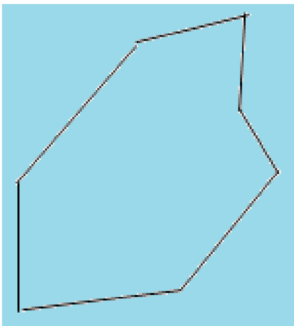
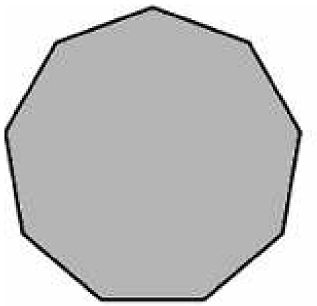
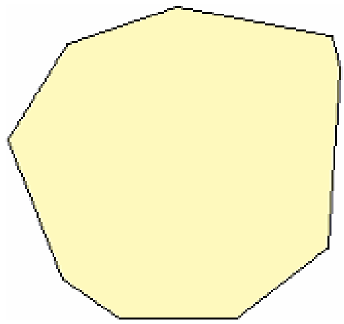
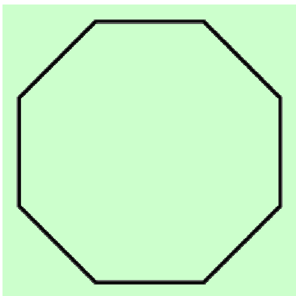
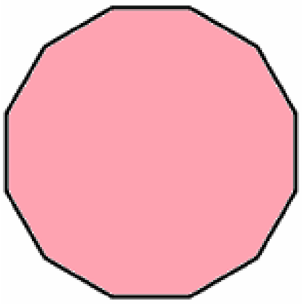
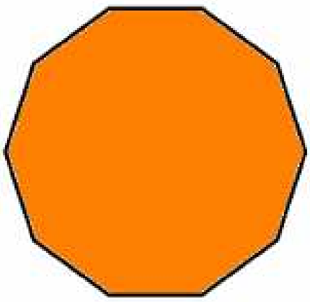
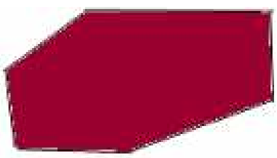
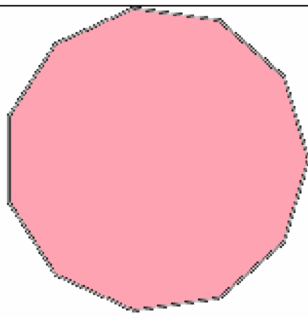
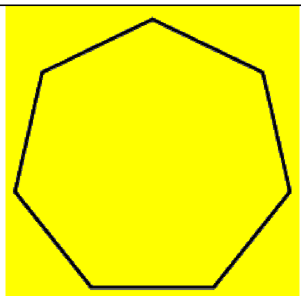
Looking at the chart discuss if the statements below are true or false.



	TRUE	FALSE
a. All rectangles are quadrilaterals.		
b. All parallelograms are rectangles.		
c. All rectangles are squares.		
d. All squares are parallelograms.		
e. A parallelogram is never a square.		
f. A square is always a rectangle.		
g. A rhombus is never a square.		
h. A trapezoid is a parallelogram.		
i. A rectangle has four right angles.		
j. A rhombus always has four equal sides.		

Activity 1

Remember how we named 2D shapes with more than four sides and write the name of the ones below. Beside the name write regular or irregular as on the example.

Regular  pentagon	 Irregular pentagon	
		
		
		

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Activity 2

In the pictures above choose a vertex and draw as many diagonals as you can from that vertex as is shown in the regular pentagon and then fill in the table below.

Name of the polygon	**How many triangles did you draw?**	**The angles add to:**	**The size of an interior angle is:**
Regular pentagon	3	3 X 180 = 520	520 : 5 = 108
Irregular pentagon	3	3 X 180 = 520	We can't say, they are all different

Activity 3

Look at the formulae below and explain why they work.

If **n** is the number of sides of a polygon, then:

- The angles of a polygon add to 180 X (n – 2) .
- In a **regular** polygon each interior angle has **180 X (n – 2)/n** degrees.

Activity 4: **Quiz**

1. This shape has:
   - 4 lines of symmetry
   - no acute angles

■ Square
■ Rectangle
■ Kite
■ Rhombus

2. This shape has:
   - one set of parallel lines
   - 4 angles
   - acute angles

■ Square
■ Rectangle
■ Trapezium
■ Rhombus

3. This shape:
   - is a quadrilateral
   - has no right angles
   - has two short lines and 2 longer lines

■ Parallelogram
■ Rectangle
■ Hexagon

■ Rhombus

4. This shape:

- has angles which add up to 360 °
- has 2 acute and 2 obtuse angles
- has 2 sets of parallel lines

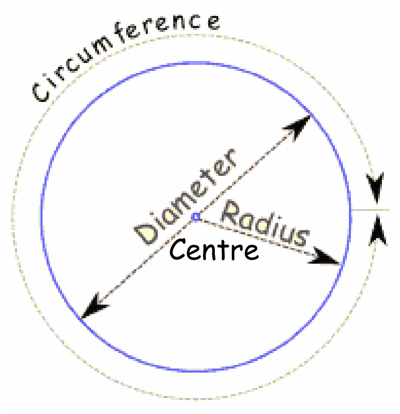
■ Equilateral triangle
■ Scalene triangle
■ Trapezium
■ Parallelogram

Make up other questions for the quiz

Activity 5: *What shape is it?*

In pairs play a guessing game: one student draws a sketch of a 2D shape and the other asks questions about its properties and makes a guess about the shape. Then swap roles.

Activity1
Look at the picture and read the definitions. Draw a circle with a diameter of 6 cm and show a diameter and the centre. Also draw another circle of 4 cm radius, show the radius and the centre.



Circle and circumference

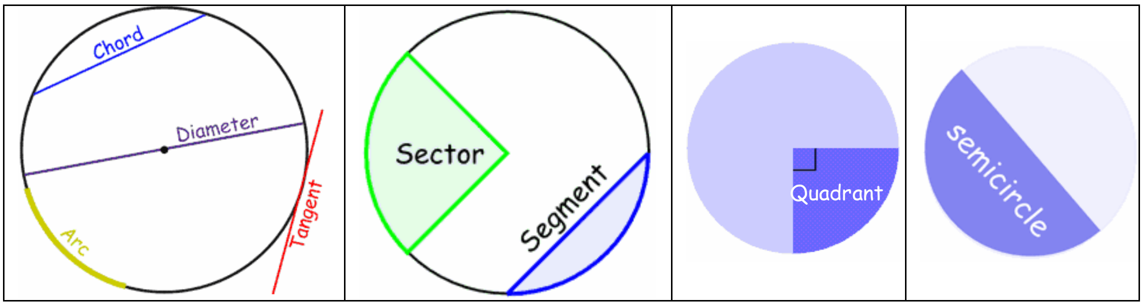
A circumference is a set of points that are a fixed distance from a **centre**".
The area inside the circumference is a **circle**

Radius and Diameter

The **Radius** is the distance from the centre to the edge.
The **Diameter** starts at one side of the circle, goes through the centre and ends on the other side.
So the Diameter is twice the Radius:
Diameter = 2 × Radius

Activity 2

Look at the pictures and match the names of the lines and slices with their definition.



A ▪	Chord	**1** ▪	Quarter of a circle
B ▪	Diameter	**2** ▪	A segment that goes from the centre to any point on the circumference.
C ▪	Arc	**3** ▪	A line that "just touches" the circle as it passes.
D ▪	Tangent	**4** ▪	A chord that passes through the centre.
E ▪	Radius	**5** ▪	A line that goes from one point to another on the circle's circumference
F ▪	Sector	**6** ▪	Half a circle
G ▪	Segment	**7** ▪	A slice of a circle made by two radius
H ▪	Quadrant	**8** ▪	A part of the circumference
I ▪	Semicircle	**9** ▪	A slice of circle made by a chord

Solution:

A	**B**	**C**	**D**	**E**	**F**	**G**	**H**	**I**

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Lesson 2

Area of 2D shapes

Activity 1

Look at these pictures and discuss in your group which one matches better with the idea of area and perimeter. Label them with the words area or perimeter.

Activity 2

a) Look at the picture of the box and answer the question.

- What can you measure in a box like this?

Think about what you need to know if you have to buy the paper for gift wrapping it, the ribbon to tie it, or if you want to fill it with sugar.

b) Match the three attributes of an object with the words given.

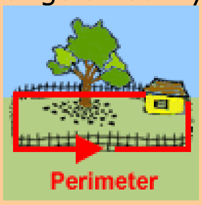
- Area
- Volume
- Perimeter

- Surrounding
- Covering
- Filling

Activity 3

Read the following definitions to see if they agree with the idea your group now has about area and perimeter and answer the questions.

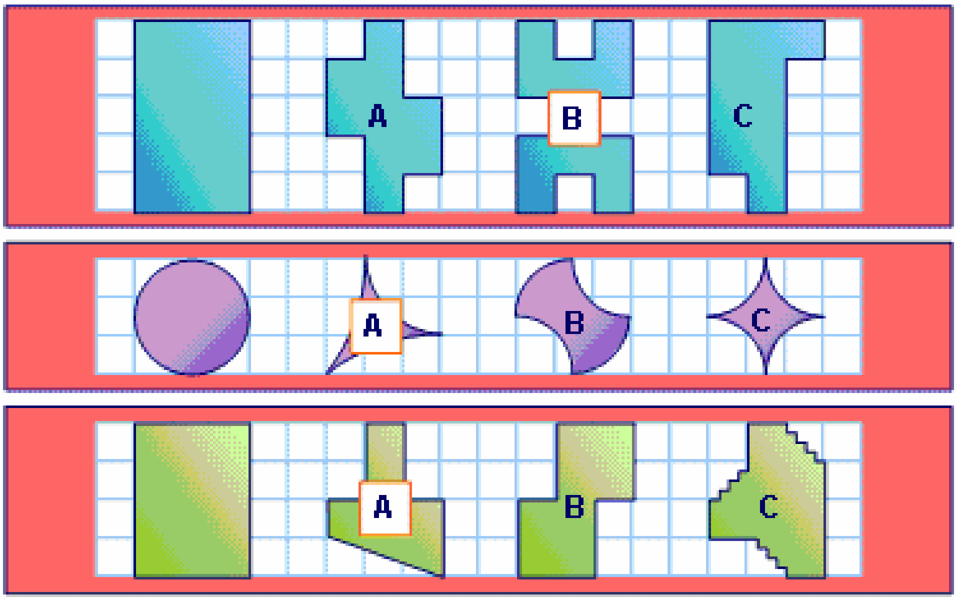
Perimeter is the distance around the outside of a shape. So, the perimeter of a polygon is the distance around the outside of the polygon. A polygon is 2-dimensional; however, perimeter is 1-dimensional and is measured in **linear units**. To help us make this distinction, look at our picture of a rectangular backyard.



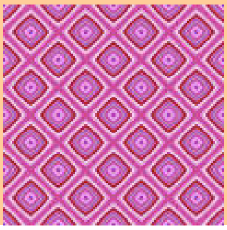
Math goodies

The yard is 2-dimensional: it has a length and a width. The amount of fence needed to enclose the backyard (perimeter) is 1-dimensional. The perimeter of this yard is the distance around the outside of the yard, indicated by the red arrow; it is measured in linear units.

a) Which shape has a different perimeter from the first shape?



Area of a shape is the region enclosed by the shape. The area of a polygon is the space inside of the polygon. Area is 2-dimensional and is measured in **square units**.

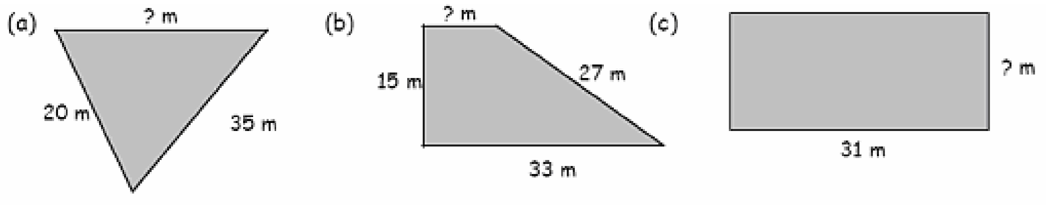


There are many practical reasons for calculating the area of a flat surface. If you want to buy carpet for a room, the **floor area** has to be calculated so the correct amount of carpet can be ordered.

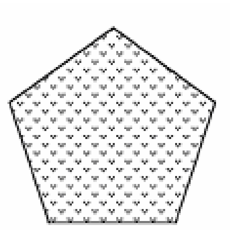
b) Write down other practical instances when you will need to know an area.

Activity 4

1. Each of these shapes has a perimeter of 80 metres. Calculate the length of the missing sides.



2. A field with perimeter 750m is in the shape of a regular pentagon. The farmer needs to replace two sides of the field with fencing costing 20 euros a meter. How much will the farmer have to pay?



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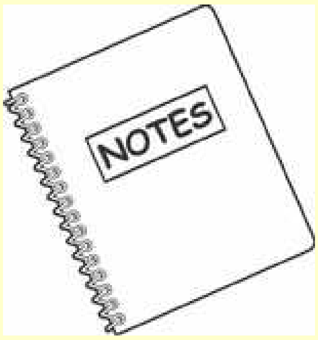
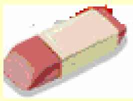
A **measurement** is a number that compares the attribute of an **object being measured** to the same attribute of a **unit of measurement**.

For example:

- If you want to measure the area of your notebook you can use a rubber.



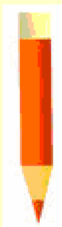


Unit of measurement

Object being measured

The area of the notebook is roughly 12 rubbers.

- If you want to measure the perimeter of your calculator you can choose using a rope or a pencil, but the measurement will be different.



Object being measured **Unit of measurement**

The perimeter of the calculator is 5 pencils or the perimeter of the calculator is 2 pieces of string.

So, it is **very important**, when finding a measurement to say which unit of measurement you have used.

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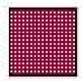
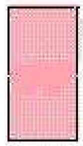
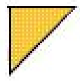
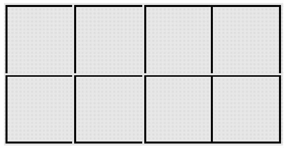
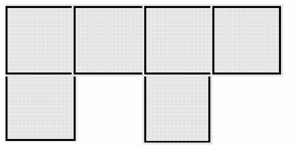
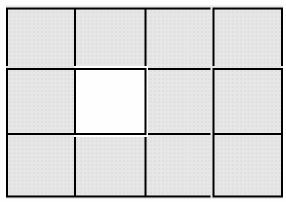
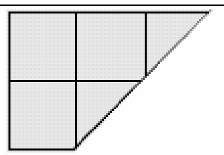
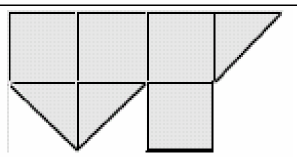
Activity 1

Write down the shaded areas of each of the following shapes using the units of measurement given:

Unit of measurement Shapes			
	8 units	4 units	16 units
			
			
			
			

Activity 2

As you have seen you can use any appropriate unit of measurement to measure the attributes of an object. In your group, discuss the following questions.

- What are the advantages or disadvantages of using your own units of measurement?
- Do all countries around the world have the same units?

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Activity 3

a) Read the text below and use the internet to find out the actual definition of the base unit of length in the SI and write it down in the box at the bottom of the page.

The International System of Units

*By the eighteenth century, dozens of different units of measurement were commonly used throughout the world. The lack of common standards led to a lot of confusion and significant inefficiencies in trade between countries. The first coherent system of units only appeared with the French revolution: the metric system. In 1960, during the eleventh Conférence Générale des Poids et Mesures (CGPM), the International System of Units, **the SI**, was developed. It now includes two classes of units : the **base units** and the **derived units**. The three most common base units in the metric system are metre, gram, and litre.*
*So, length, for example, is measured in **metres** in the metric system although if you are measuring the length of your finger or the length of the Nile River, you will use different subunits of the metre. The **subunits** are used when measuring very large or very small things. It wouldn't make sense to measure your finger length in metres, the unit is too big. Neither would you express the length of the Nile River in metres, the unit is too small.*
*The derived units are those formed by combining base units. For example the derived unit for measuring a surface is **square metre** and the one for volumes is the **cubic metre**.*

The base unit of length

A meter is

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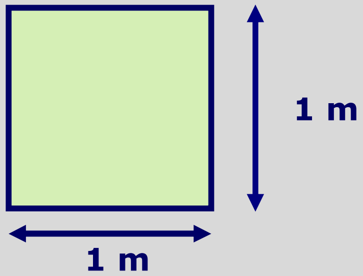
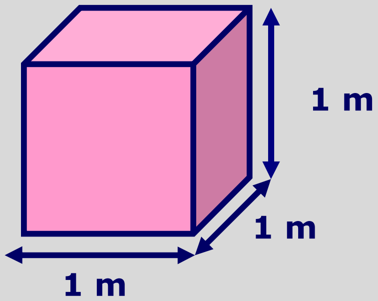
The derived units of the metre

	
The **square metre** (**m^2**) is the SI derived unit of surface. It is the surface of a square with sides one metre in length.	The **cubic metre** (**m^3**) is the SI derived unit of volume. It is the volume of a cube with edges one metre in length

b) Read the text below and use the internet to find out some imperial units (surface, lengths, weights, capacity) still used in the UK. Write down their names, a rough idea of their conversion to SI units and some examples of use.

Imperial units or the **imperial system** is a system of units, first defined in the British *Weights and Measures Act* of 1824, later refined (until 1959) and reduced. Systems of imperial units are sometimes referred to as foot-pound-second, after the base units of length, mass and time. The units were introduced in the *British Empire*, excluding the then already independent United States. As of 2008, all countries that used the imperial system have become officially *metric* (except for Burma and Liberia), but imperial units continue to be used alongside metric units. Imperial units of length are commonly used, for example, in British cars the speedometer measures speed in miles per hour. Also, most people still give their heights in feet and inches and their weight in stones and pounds.
Also in United States they still use the **U.S. customary units** but there are many points of difference between the two systems.

Activity 1

Answer the questions, and afterwards check the answer in the table below. The table shows all the lengths' subunits, their abbreviations and some conversions.

a. How many **millimetres** are there in a **centimetre**?

There are ____________ millimetres in a centimetre.

b. How many centimetres are there in a metre?

c. How many **decimetres** are there in a metre?

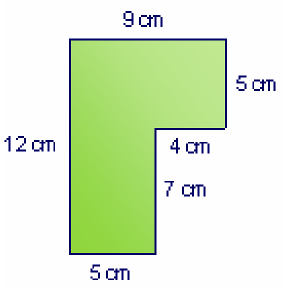
d. How many metres are there in a **kilometre**?

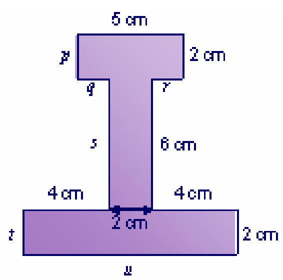
Subunits of Length

10 millimetres (mm)	**= 1 centimetre (cm)**
10 centimetres	**= 1 decimetre (dm) = 100 millimetres**
10 decimetres	**= 1 metre (m) = 100 centimetres = 1000 millimetres**
10 metres	**= 1 decametre (dam)**
10 decametres	**= 1 hectometre (hm) = 100 metres**
10 hectometres	**= 1 kilometre (km) = 1000 metres**

Activity 2

Find the perimeter of the shapes and express it in millimetres.





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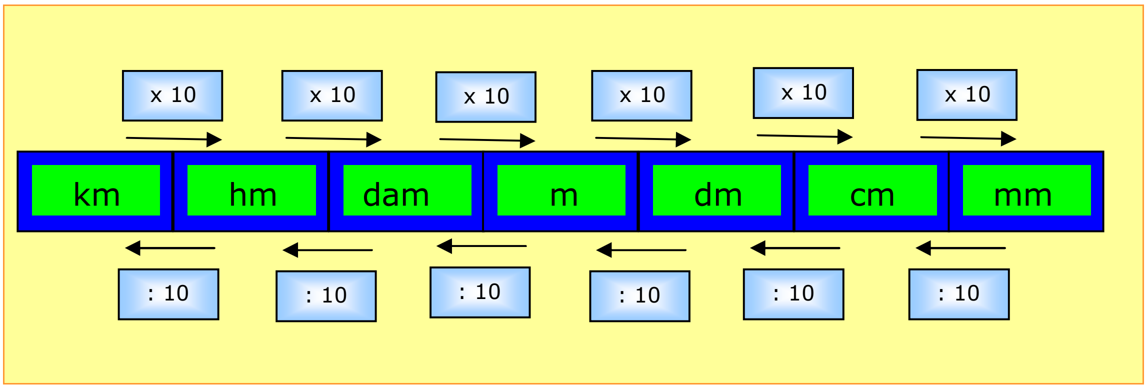
Activity 3

Look at this picture and discuss in your group how to use it and write some examples. Afterwards work out the exercises.



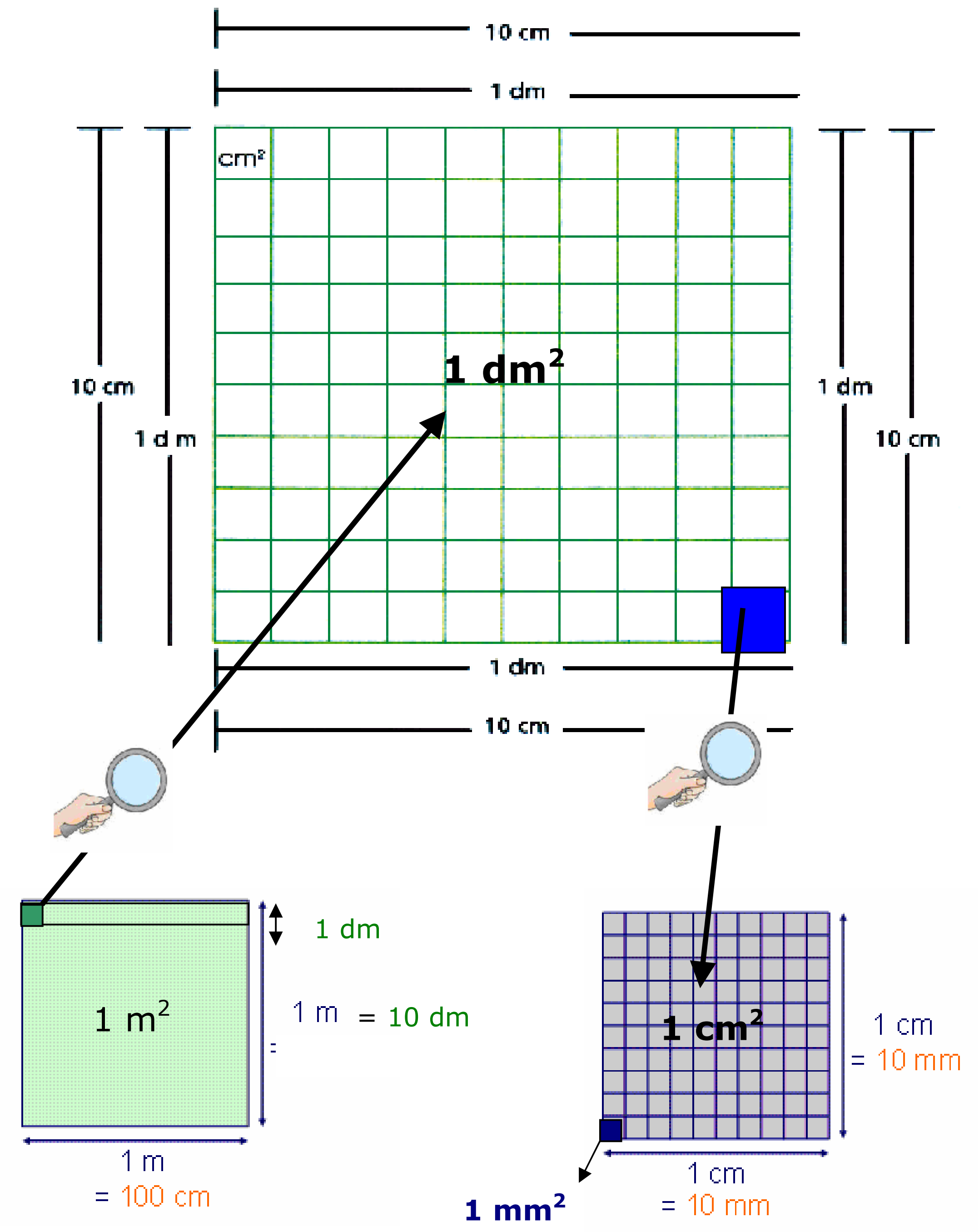
Examples:

a. Look at the sign. How many kilometres to Glasgow?

b. A 6 metre length of wood is cut in three places, so that all the pieces are of the same length. What is the length of each piece in millimetres?

Activity 4

Look very carefully at these pictures and answer the questions. Afterwards fill in the gaps in the conversion table.



a. What is a square metre?

b. What is a square centimetre?

c. What is a square millimetre?

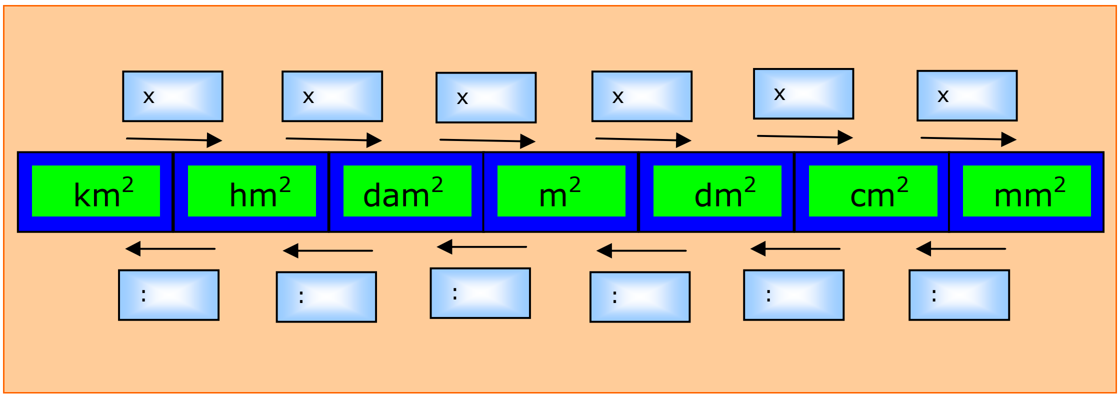
d. How many **square millimetres** are there in a **square centimetre**?

There are ___________ square millimetres in a square centimetre.

e. How many **square centimetres** are there in a **square decimetre**?

f. How many **square decimetres** are there in a **square metre**?

g. How many **square centimetres** are there in a **square metre**?



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Activity 5

Convert the following measurements to the indicated unit as shown in the example.

a. $2\ km^2 = 2 \cdot 1000000 = 2000000\ m^2$

b. $35\ mm^2 = 35 : 100 = 0{,}35\ cm^2$

c. $135\ m^2 =$ $=$ cm^2

d. $315\ dam^2 =$ $=$ hm^2

e. $3{,}75\ dm^2 =$ $=$ mm^2

f. $31{,}235\ km^2 =$ $=$ dam^2

g. $935{,}6\ dam^2 =$ $=$ km^2

h. $35000\ m^2 =$ $=$ km^2

i. $0{,}00635\ m^2 =$ $=$ cm^2

j. $0{,}789315\ m^2 =$ $=$ mm^2

k. $3578{,}96\ m^2 =$ $=$ hm^2

Activity 6

Identify the units of measurement that would be appropriate for measuring:

a) the diameter of a basketball.

A. centimetres B. square inches C. cubic millimetres D. cubic inches

b) the amount of propane in a cylindrical propane tank.

A. square yards B. meters C. centimetres D. cubic feet

c) the distance around a rectangular garden plot.

A. square feet B. meters C. cubic centimetres D. cubic yards

d) the amount of material needed to cover a rectangular wall.

A. square yards B. cubic feet C. millimetres D. meters

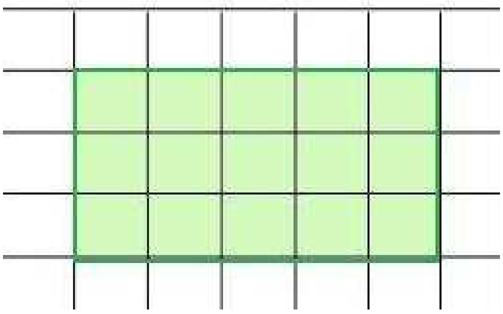
e) the amount of flat material needed to construct a rectangular box.

A. meters B. cubic meters C. cubic feet D. square inches

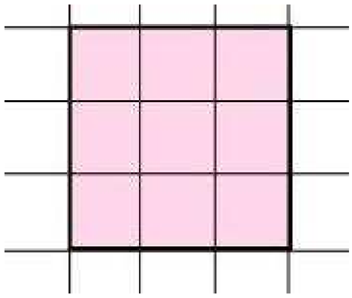
Activity 1

Look at the shapes below and find the area by counting the square centimetres they have. Can you find a formula that helps to find these areas?

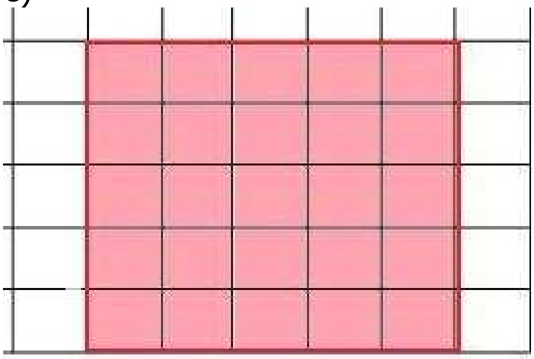
a)



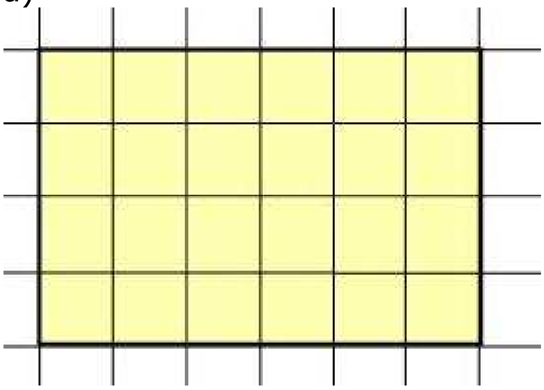
b)



c)



d)

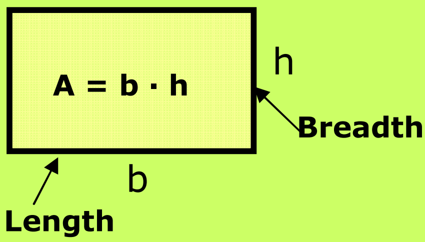


Area of the rectangle and the square

The area of a rectangle is given by the formula :

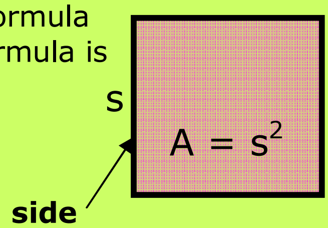
Area = Length · Breadth

(Area equals length times breadth)



As a square is also a rectangle we can use the same formula But since the length and breadth are the same, the formula is usually written as:

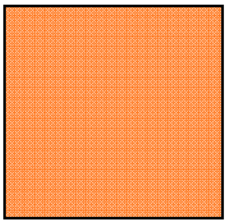
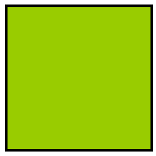
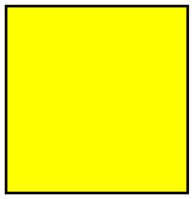
Area = side · side = s^2



(In strictly correct mathematical wording the formula above should be spoken as "s raised to the power of 2", meaning s is multiplied by itself. But we usually say it as "s squared").

Activity 2

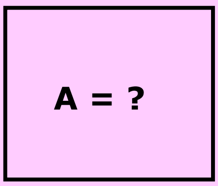
Find the area of the following squares. Follow the example.

<table>
<tr>
<td rowspan="2">a) s = 6 cm

1st step: Write down the area formula.
$A = s^2$
2nd step: Substitute the dimensions in the formula and do the sum.
$A = 6^2 = 36\ cm^2$
* Don't forget to write down the units in the answer.</td>
<td>b)

3 cm</td>
</tr>
<tr>
<td>c)
5 cm
</td>
</tr>
</table>

Activity 3

Look at the example and follow the steps to solve the exercise.

Example: The sides of a square are 10 cm long. What is its area?



s =10 cm

1st step: Draw a diagram showing the shape with the known measures and the asked ones?

2nd step: State the formula.

$$A = s^2$$

3rd step: Substitute the dimensions on the formula and do the sum.

$$A = 10^2 = 100\ cm^2$$

*** Don't forget to write down the units in the answer.**

The sides of a square are 8 cm long. What is its area? What is its perimeter?

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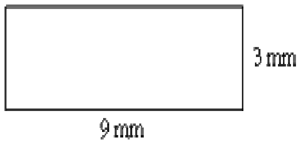
Activity 4

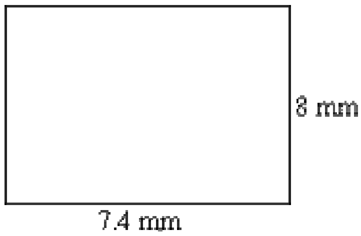
Find the area of the following rectangles.

a)

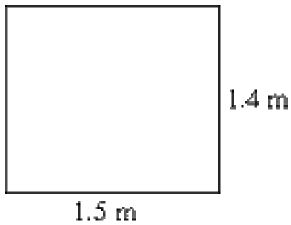


b)

c)

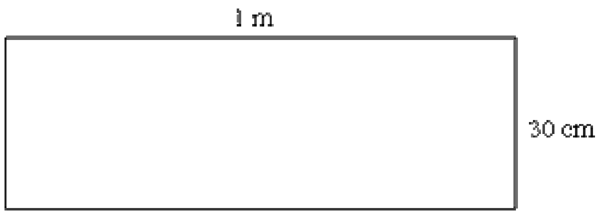


d)



Activity 5

Look at the rectangle below.



(a) Find the area of the rectangle in cm² by converting the length to cm first.

(b) Find the area of the rectangle in m² by converting the breadth to m first.

(c) Find the perimeter of the rectangle in cm and convert it in m.

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Activity 6

Solve the following questions. Remember to follow the steps.

(a) The length of a rectangle is 5 cm long and its breath is 3 cm long. What is its area? What is its perimeter?

(b) The length of a rectangle is 70 mm long and its breadth is 4 cm long. What is its area? What is its perimeter?

Activity 7

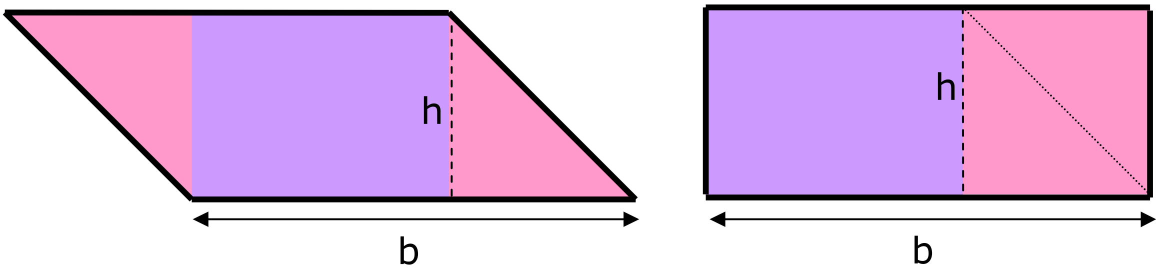
Solve the following questions. Remember to follow the steps

(a) A rectangle has an area of 48 cm^2. The length of one side is 6 cm. What is the perimeter of the rectangle?

(b) The backyard of my house has an area of 36 m^2. The length of one side is 900 cm. What is the breath of the field?

Activity 1

Look at the shapes and answer the questions. Afterwards read the box below.



a. What shapes are they?

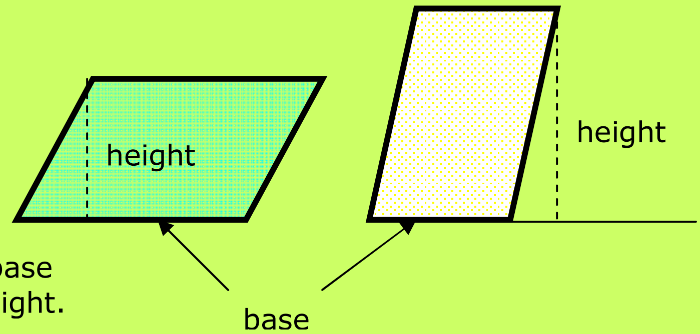
b. Which one has a bigger area? Why?

c. What formula can you use to find the area of a parallelogram?

Area of the parallelogram

The area of a parallelogram is given by the formula:

Area = b · h



where **b** is the length of any base
and **h** is the corresponding height.

Recall that any side can be chosen as the base. You must use the height that goes with the base you choose. The height of a parallelogram is the perpendicular distance from the base to the opposite side (which may have to be extended).

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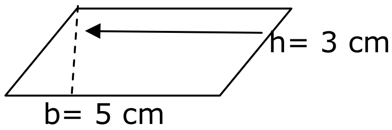
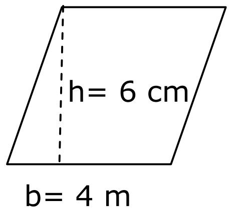
Activity 2

Find the area of the following parallelograms.

a)



b)



Activity 3

Solve the problems following the steps.

a. Find the area of a parallelogram with a base of 8 cm and a height of 3 cm.

b. Find the area of a parallelogram with a base of 4 metres and a height of 9 metres

c. A parallelogram has an area of 54 square centimetres and a base of 6 centimetres. Find the height.

d. A parallelogram-shaped garden has an area of 42 square yards and a height of 6 yards. Find the base.

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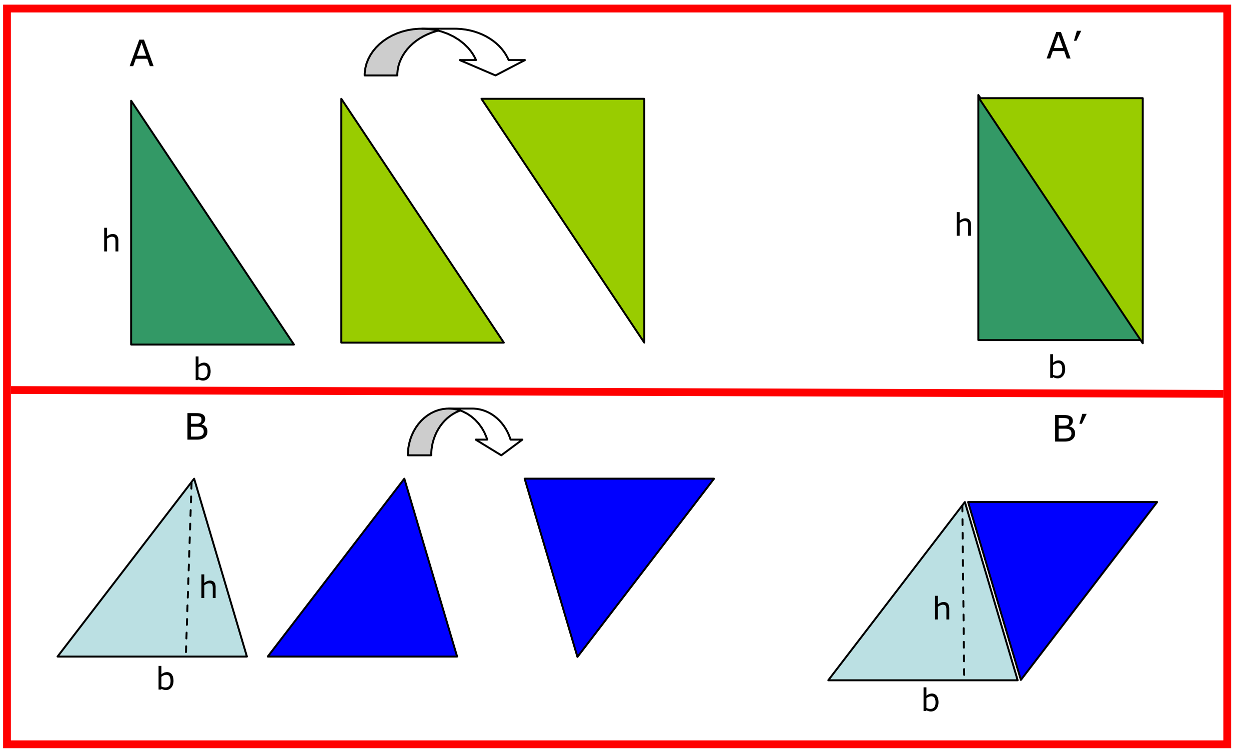
Activity 1

Look at these pictures and find out the formula of the area of a triangle.



Here there are some questions that will help you.

- What is the relationship between shapes A and A′?
- What is the formula of the area of A′?

  So, area of A is:

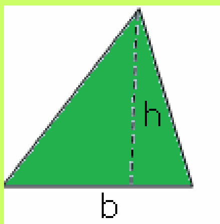
- What is the relationship between shapes B and B′?
- What is the formula of the area of B′?

  So, area of B is:

Area of the triangle

The area of a triangle is given by the formula:

$$\textbf{Area} = \frac{\mathbf{b \cdot h}}{\mathbf{2}}$$



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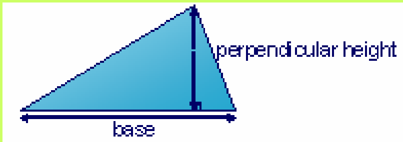
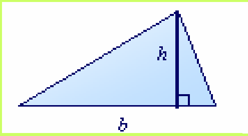
Activity 2

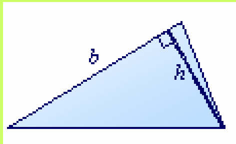
Read the following and explain with a diagram what happens in a right angled triangle and in an isosceles triangle.

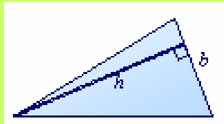
Heights of a triangle.



Any side of the triangle can be taken as the base, as long as the height is perpendicular to it



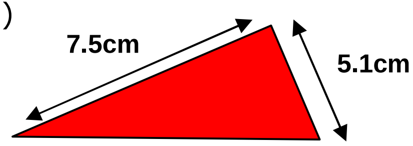




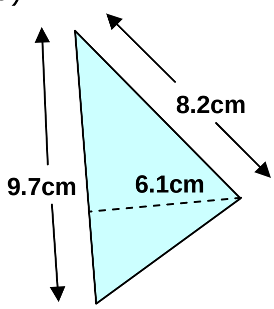
Activity 3

Find the area of the following triangles. Remember to write the formula first.

a)



b)



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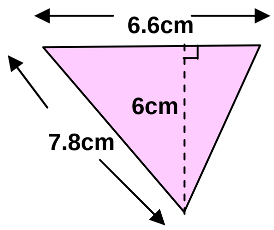
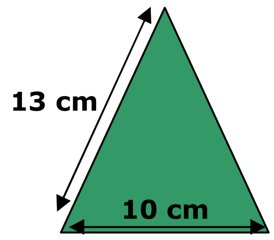
c)



d)



Activity 4

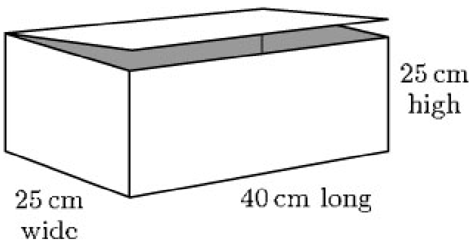
Design a real sized logo for your group using only parallelograms (including squares and rectangles) and triangles and find out how much material will you need to make it.

Activity 1

A friend of yours decides to lay lawn in his garden, which measures 7 m by 5 m, but he wants to leave two rectangular areas, each 2 m by 1 m, for flowerbeds. What area of lawn will be needed?

Activity 2

A girl is decorating a box by gluing wrapping paper on each face. She wants to put paper on the sides, the top and the bottom, and intends to cut out six pieces of paper and stick them on. Assuming no wastage, calculate what area of paper she will need.



Activity 3

A rug measures 3 m by 2 m. It is to be laid on a wooden floor that is 5 m long and 4 m wide. The floorboards not covered by the rug are to be varnished. What area of floor will need to be varnished? If a tin of varnish covers 2.5 m^2. How many tins will be required?

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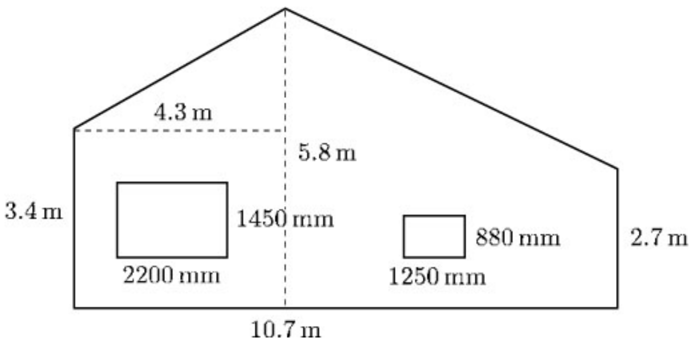
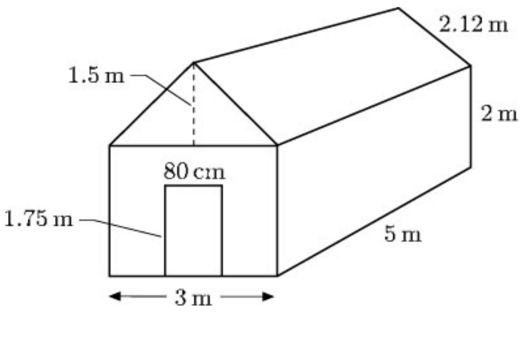
Activity 4

This diagram represents the end wall of a bungalow; the wall contains two windows. The wall is to be treated with a special protective paint. In order to decide how much paint is required, the owner wants to know the area of the wall. Divide the wall up into simple shapes and then find the total area.



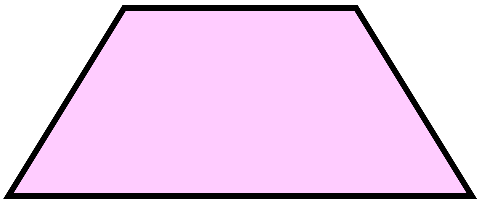
Activity 5

The diagram below shows the dimensions of a frame tent. Calculate the amount of canvas needed to make the tent, ignoring the door which is made of different material.



Activity 1

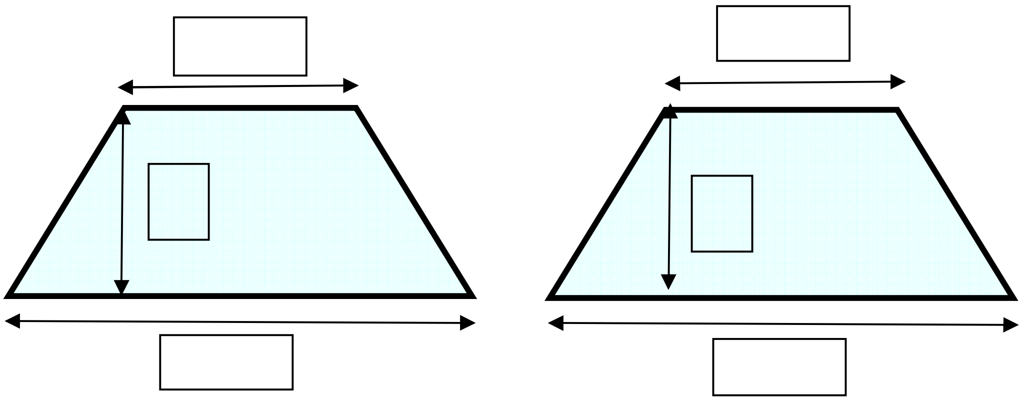
Read the definitions of base and height of a trapezium and draw and label them on the shape below. Use B for the longest base, b for the shortest and **h** for the height.

• The **base** of the trapezium is any of the two parallel sides. • The **height** of a trapezium is the perpendicular distance from the two parallel sides	

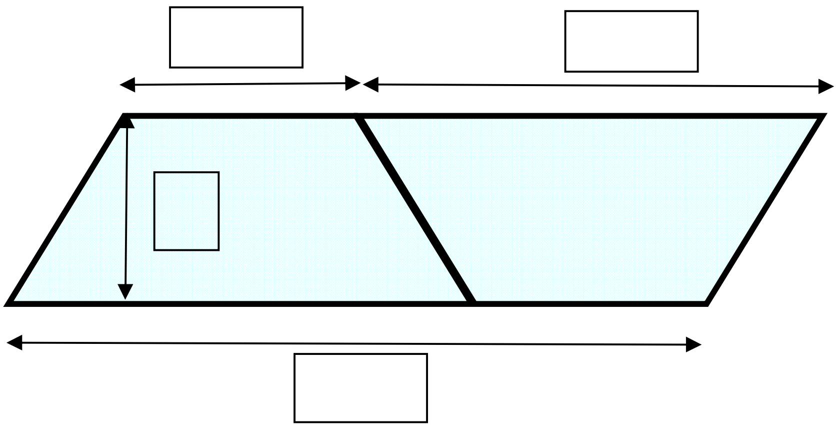
Activity 2

If **b** and **B** are the lengths of the two parallel bases of a trapezoid, and **h** is its height, what is the area of the trapezoid? Fill in the boxes with the suitable letter on steps 1 and 2 and answer the questions on step 3.

1. Consider two identical trapezoids and label the arrows:



2. "Turn" one around and "paste" it to the other along one side. Label the arrows.



3. The shape formed is a parallelogram. Answer the questions.

- What is the relationship between the area of the parallelogram and the area of the trapezium?

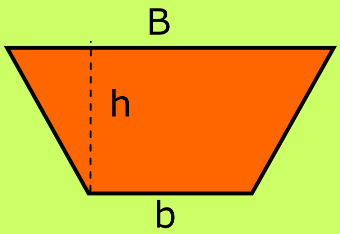
- What is the formula of the area of the parallelogram?

- What is the formula of the area of the trapezium?

Area of the trapezium

The area of a trapezium is given by the formula:

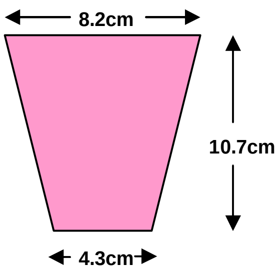
$$\textbf{Area} = \frac{(B+b)\cdot h}{2}$$



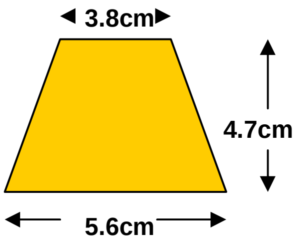
Activity 3

Find the area of the two trapezia below.

a)

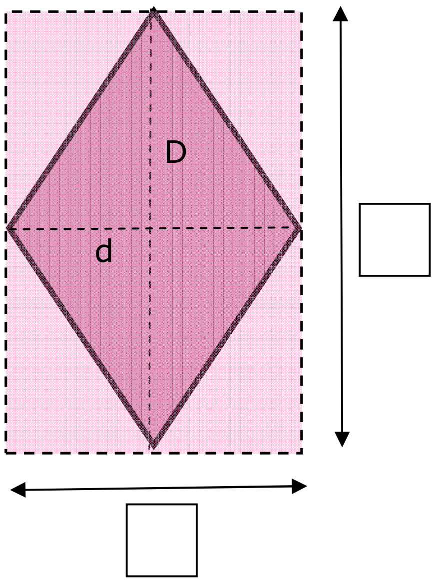


b)



Activity 1

Look at the picture; it's formed by a rhombus inside a rectangle. D and d are the diagonals of the rhombus. **D** is the big one and **d** the small one.



a) Explain the relationship between the area of the rhombus and the area of the rectangle.

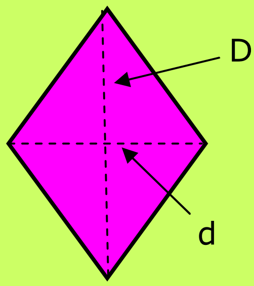
b) Write the dimensions of the rectangle and the formula of its area.

c) Write the formula of the area of the rhombus.

Area of the rhombus

The area of a rhombus is given by the formula:

$$\textbf{Area} = \frac{\mathbf{D \cdot d}}{\mathbf{2}}$$



Activity 2

Use a similar caculation to the one used in *activity 1* and find out the formula for the area of a kite.

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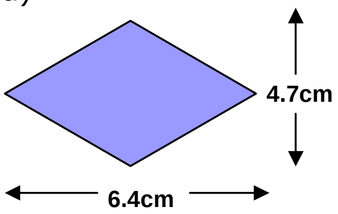
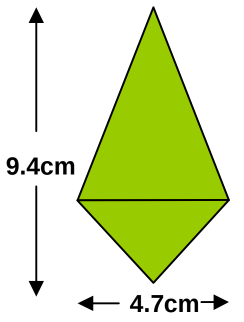
Activity 3

Find the area of the following shapes. Remember to write the formula first.

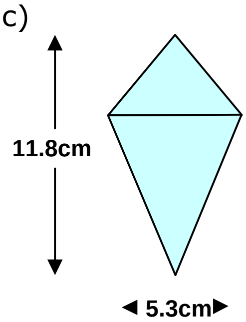
a)



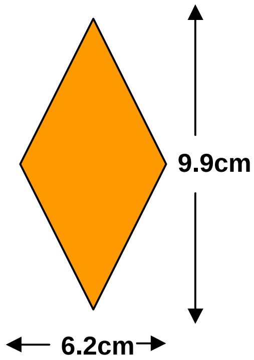
b)



c)



d)

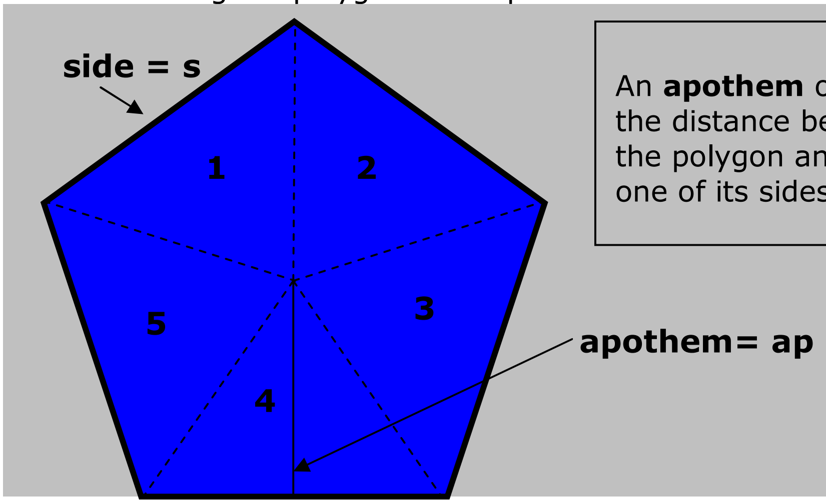


Activity 4

We want to build a kite. We have 2 pieces of wood for the skeleton that measure 60 cm and 80 cm. How much fabric do we need?

Activity 1

Look at this regular polygon. It is split into five isosceles triangles.



An **apothem** of a regular polygon is the distance between the centre of the polygon and the middle point of one of its sides

a. Write the formula of any of the triangles using the letters on the diagram above.

b. Find out the formula of the area of a regular pentagon.

c. Draw a regular hexagon, split it into triangles and letter it as showed. Which is the area of a regular hexagon?

d. Repeat section ***c*** using a regular octagon.

Activity 2

Explain why the following formula is equivalent to the ones you found out in activity 1.

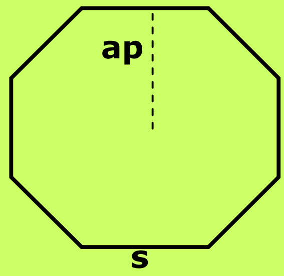
Area of a regular polygon

The area of a regular polygon is given by the formula:

$$\textbf{Area} = \frac{P \cdot ap}{2}$$

P is the perimeter of the polygon.

P = length of side X number of sides



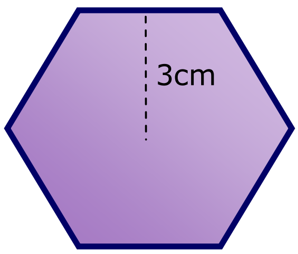
If n = number of sides, then

$$P = n \cdot s$$

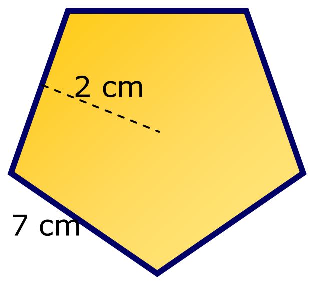
Activity 3

Find the area of the following regular polygons. Remember to state the formula.

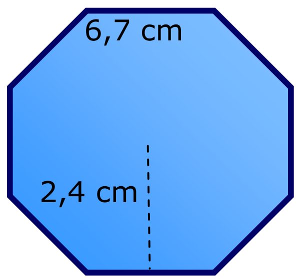
a)



b)



c)



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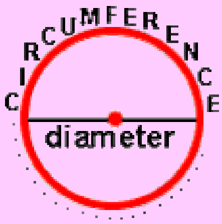
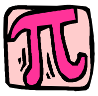
Things to remember

The distance around a circle is called the **circumference** and the distance across a circle through the centre is called the **diameter**.

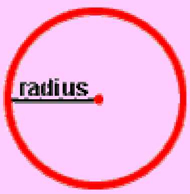


π is the ratio of the circumference of a circle to the diameter. Thus, for any circle, if you divide the circumference by the diameter, you get the same value. This relationship is expressed in the following formula:

$$\frac{C}{d} = \pi$$



This symbol is the Greek letter pi. It stands for a number that can never be found exactly but you will find a good approximation if you press the **π** button on your calculator. We will use this value to find out the area and the perimeter of a circle.



- The formula to find **the circumference** of a circle is:

$$C = 2\pi r$$

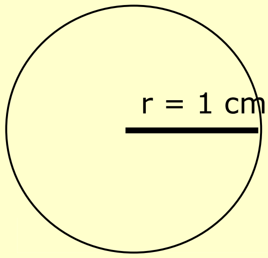
- The formula to find the **area** of a circle is:

$$A = \pi r^2$$

Activity 1

Calculate the circumference of each of the following circles. Look at the steps in the example.

a)



1st step. Write the formula

$$C = 2\pi r$$

2nd step: Substitute the values

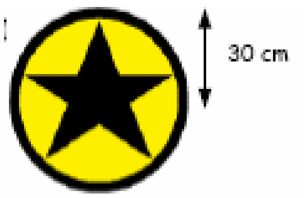
$$C = 2\pi\ 1$$

3rd step: Do the sum and round the answer to two decimal points. Write the units.

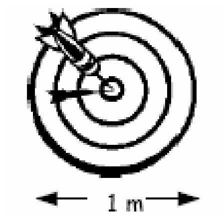
$$C = 6{,}28 \text{ cm}$$

b)

c)



d)



Activity 2

A satellite orbits 900 km above the earth.
Assuming the radius of the earth is 6350 km, calculate the distance the satellite travels in one orbit.

Activity 3

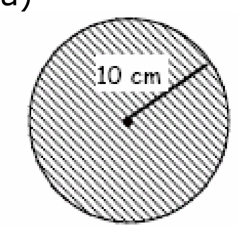
This CD has an outer circumference of 40 centimetres. The hole has a 0.5 centimetre radius.
a) Calculate the radius of the CD.

b) Calculate the circumference of the hole.

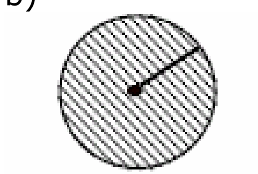
Activity 4

Calculate the area of each circle below : (You **should** set down 3 lines of working)

a)

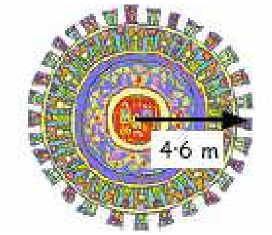


b)



Activity 5

Calculate the area of the circular carpet shown. It has a radius of 4·6 metres.



Activity 6

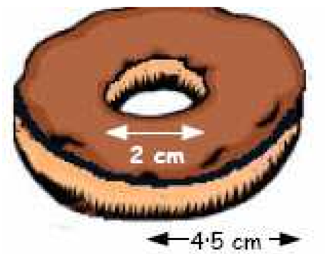
Work out the area of this coloured counter which has diameter 1·8 metres.

Activity 7

A circular trampoline has a circumference of 10·99 m. Calculate its area, to the nearest m2.

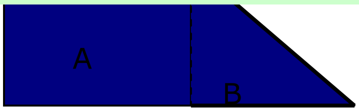
Activity 8

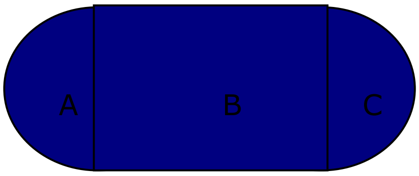
This doughnut has an outer radius of 4·5 cm and the hole in the centre has a diameter of 2 cm. Calculate the area of chocolate required to cover the top part of the doughnut.



A compound shape is a shape made up of two or more different shapes. To find the total area of a compound shape we just find the area of each part and add them together. For example:

Total area = Area of A + Area of B



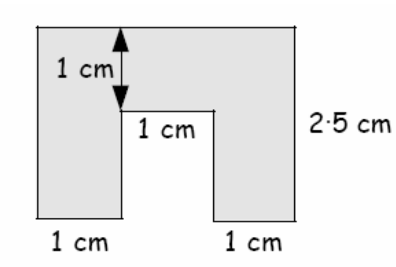


Here there are 2 semi-circles. A and C. Together they make a circle.
Total area = Area of circle (A and C) + Area B

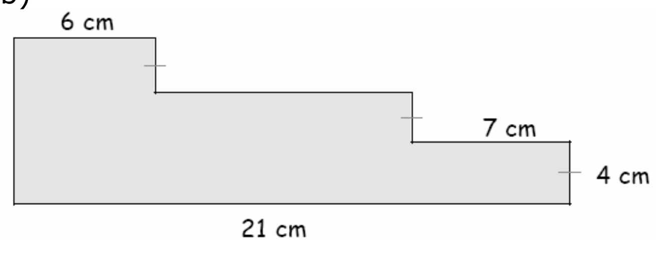
Activity 1

Find the area of the following compound shapes.

a)



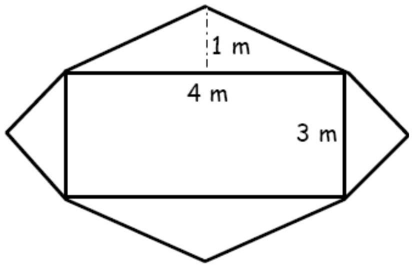
b)



Activity 2

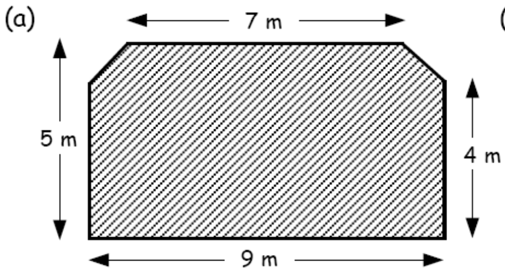
A company logo uses a rectangle (4 metres by 3 metres) and two pairs of isosceles triangles, each with a height of 1 metre, as shown.

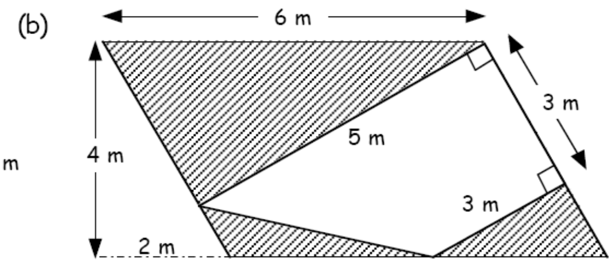
Calculate the total area of the logo.



Activity 3

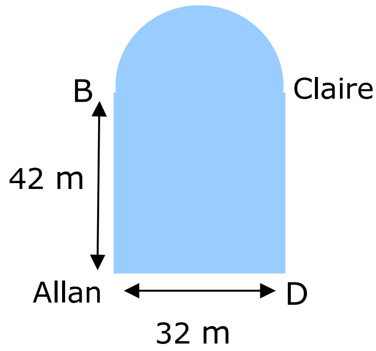
Calculate the shaded area of each of the following compound shapes.





Activity 4

A boating pond, in Pollard Park consists of a rectangle with a semi-circular end as shown.
Allan and **C**laire are at opposite ends of the pond.
Allan walks round the pound to meet Claire.
How much further would he walk if he went from A to B round to C instead of A to D to C?



Activity 5

Draw a nice compound shape, make up and write down the measurements needed to find out the area and the perimeter and do so. When you finish swap it with your partner and ask him/her to calculate the area. Afterwards you will check each others answers together.

Lesson 3

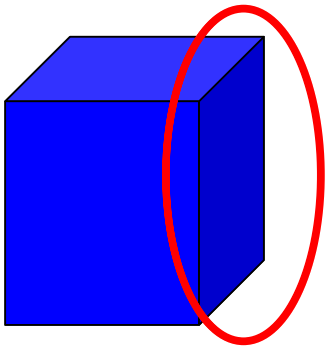
3D shapes: Introduction, classification, properties.

- **Worksheet 1:** Introduction and classification
- **Worksheet 2:** Polyhedrons
- **Worksheet 3:** Solids of revolution

Activity 1

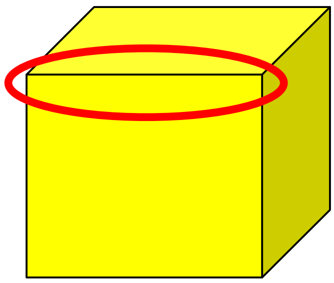
A 3D shape is a shape with three dimensions: length, width and height. They are also called solids. Look at the following pictures and match each part of a 3D shape with its definition.

a. Face



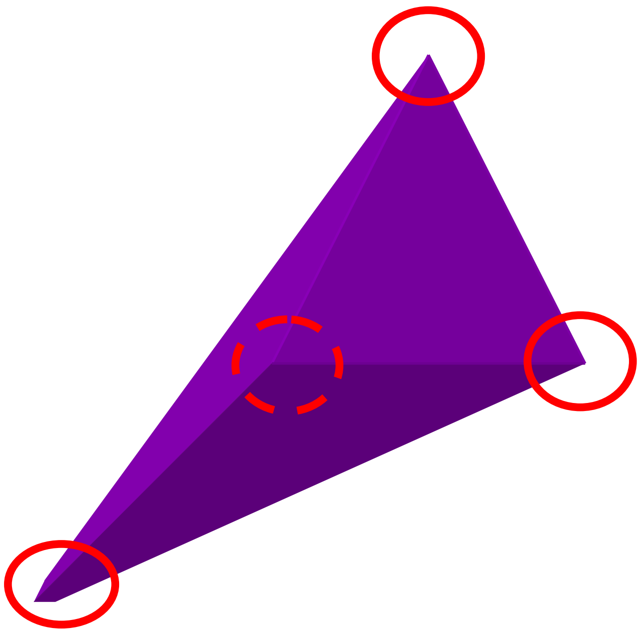
This shape has six faces

b. Edge



This shape has twelve edges

c. Vertex

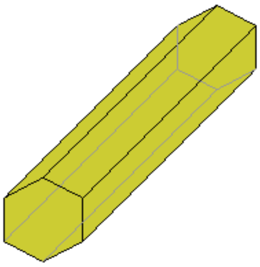
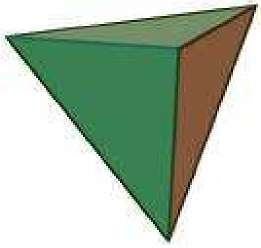
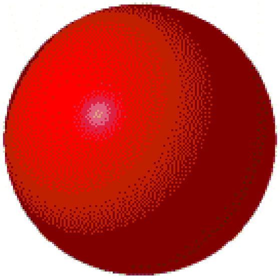
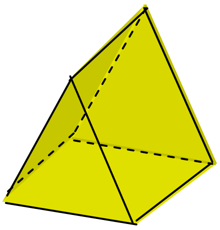
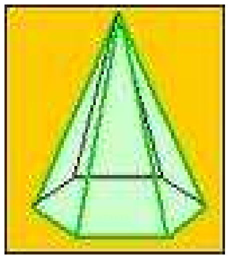
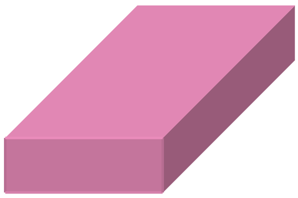
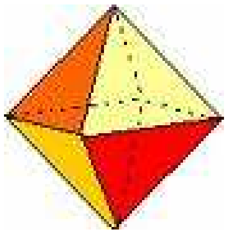
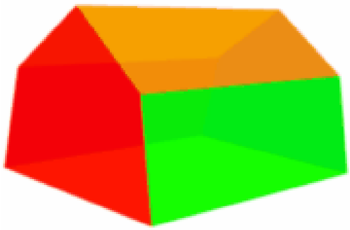
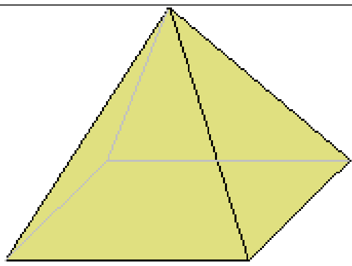
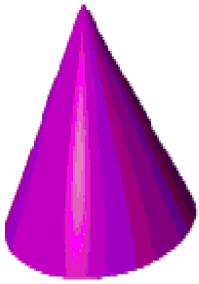


This shape has four vertices

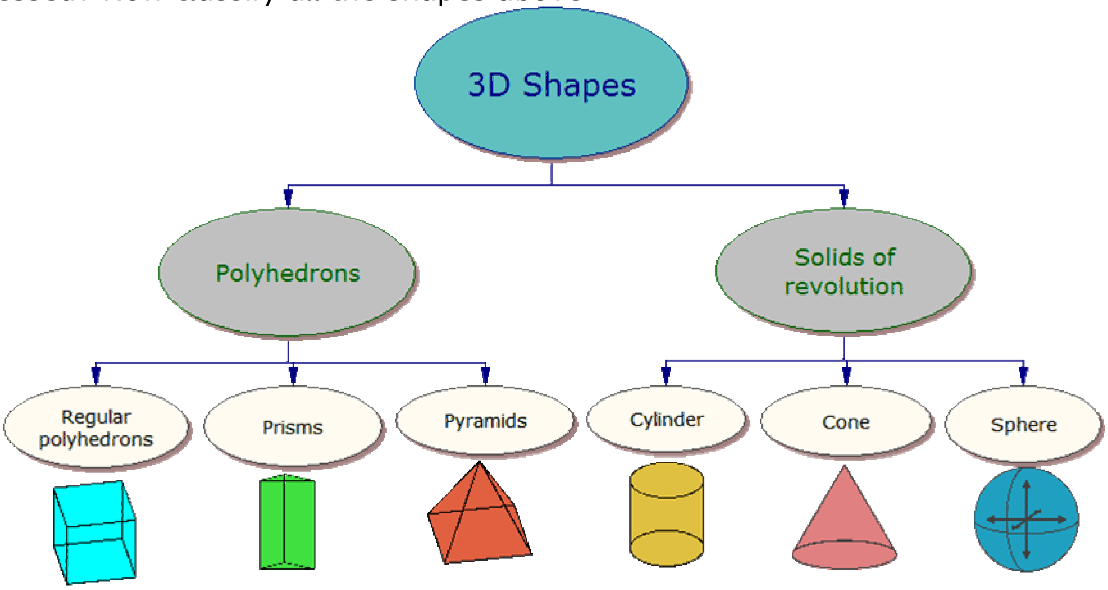
(1)	The place where three or more *edges* meet.	a)	Face
(2)	Part of a shape that is flat.(Or curved)	b)	Edge
(3)	The line where two *faces* meet.	c)	Vertex

Activity 2 Look at all these 3D shapes and discuss in your group which is the best way of classifying them. Answering the following questions may help:

- Are its faces flat or curved?
- Does it have two parallel faces?
- Are all its faces the same polygon?
- Which kind of polygons are its faces?
- ....

When you have finished look at the following diagram. Is this what you have guessed? Now classify all the shapes above.



I. Polyhedrons

A three-dimensional shape whose faces are polygons is known as a polyhedron. This term comes from the Greek words poly, which means "many," and hedron, which means "face." So, quite literally, a polyhedron is a three-dimensional object with many faces.

A. Regular polyhedrons

A three-dimensional shape each face of which is a regular polygon with equal sides and equal angles.

B. Prisms

A prism is a polyhedron with two faces that are congruent polygons (they're the same size and shape), and other faces are parallelograms.

C. Pyramids

A pyramid is a polyhedron with a face that is a polygon and all the other faces are triangles

II. Solids of revolution

A three dimension shape generated by revolving a given curve around a line.

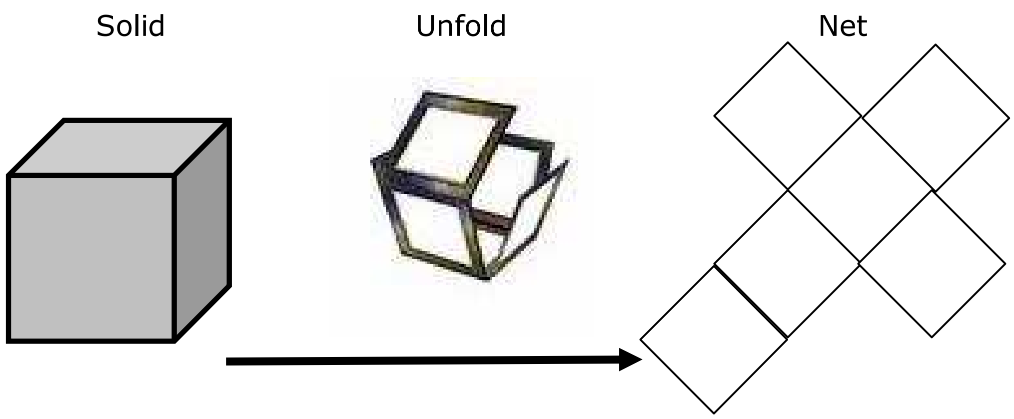
A. Cylinder **B. Cone** **C. Sphere**

Activity 3

A die is in the shape of a cube. A shoe box is in the shape of a rectangular prism. A Toblerone box is in the shape of a triangular prism. These shapes are all examples of polyhedrons.
Write down other examples of objects and which 3D shapes they are.

Activity 4

The **net** of a 3D shape is a representation of its faces in two dimensions. The net is what appears when the solid is unfolded. For example:

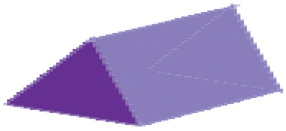


Look at the pictures in the following page and match each solid with its net.

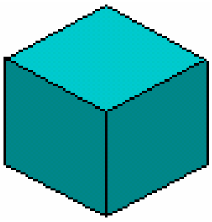
Solid	A	B	C	D	E
Net					

Worksheet 1. Introduction and classification

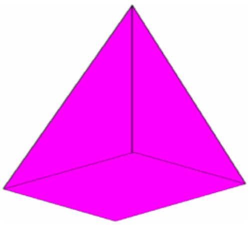
A



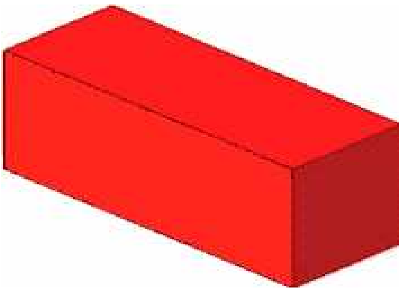
B



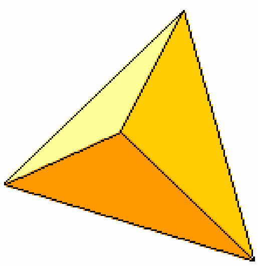
C



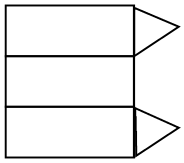
D



E



1



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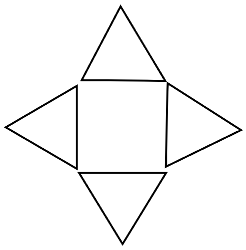

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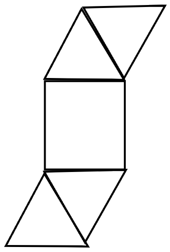

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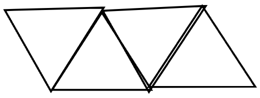

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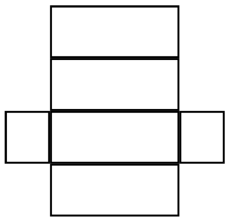
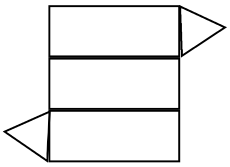
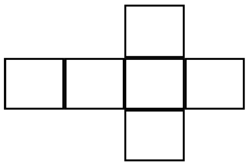


6



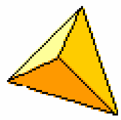
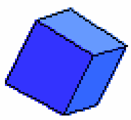
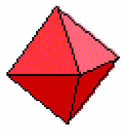
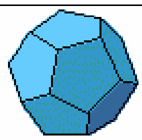
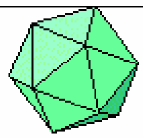
7

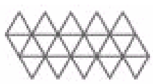
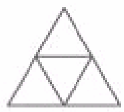


Activity 1

A regular polyhedron, also called a Platonic solid is a three-dimensional shape each face of which is a regular polygon with equal sides and equal angles. Every face has the same number of vertices, and the same number of faces meets at every vertex. There are only five platonic solids

Fill the box with the description and the letter of the net that matches the shape shown:

Name	Picture	Description	Net
Tetrahedron			
Cube or hexahedron			
Octahedron			
Dodecahedron			
Icosahedron			

A

B

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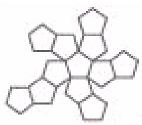
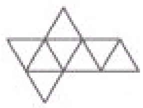
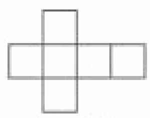
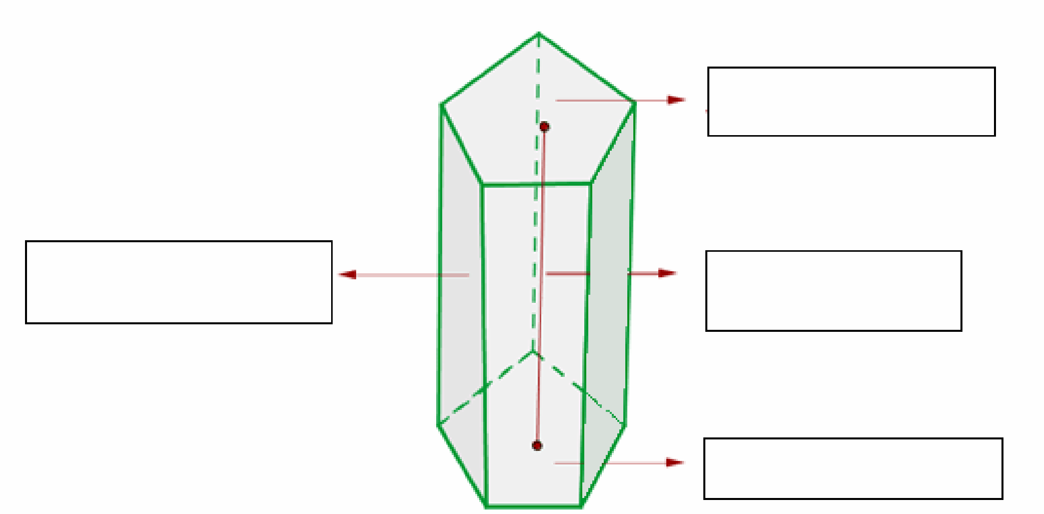

D

E


- A three-dimensional shape with 12 regular pentagonal faces, 20 vertices and 30 edges.
- A three-dimensional shape with 8 equilateral triangular faces, 6 vertices and 12 edges.
- A three-dimensional shape with 4 equilateral triangular faces, 4 vertices and 6 edges.
- A three-dimensional shape with 20 equilateral triangular faces, 12 vertices and 30 edges.
- A three-dimensional shape with 6 squared faces, 8 vertices and 12 edges.

Activity 2

a) Read the box below and use the names in bold to label the elements of the prism.

A prism is a polyhedron for which the top and bottom faces (known as the **bases**) are congruent (have the same shape and size) polygons, and all other faces (known as the **lateral faces**) are parallelograms.
The two bases of a prism are parallel and the distance between them is called the **height** of the prism



b) Read the box and name the prisms shown. Write down if they are right prisms or not.

When the lateral faces are rectangles, the shape is known as a **right prism.**

When the lateral faces are parallelograms but not rectangles we call them **non-right prism**.

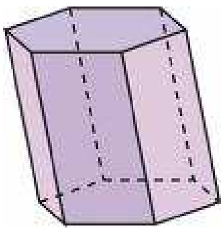
A prism is named by the shape of its base. For instance, a triangular prism has bases that are triangles, and a pentagonal prism has bases that are pentagons.

Some prisms have particular names:

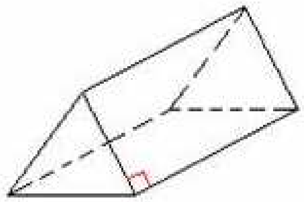
- A **parallelepiped** is a prism with six faces, all parallelograms.

- A **cuboid** is a prism with six faces, all rectangles. If at least two of the faces are squares it can also be called **square prism** (This doesn't stop it to be called **rectangular prism**). And if all the faces are squares then is a **cube**. So a cube is just a special case of a square prism, and a square prism is just a special case of a rectangular prism. And they are all cuboids.

(1)



(2)



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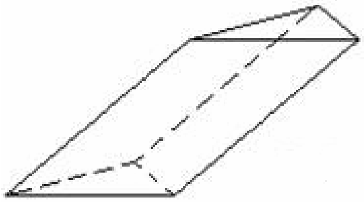

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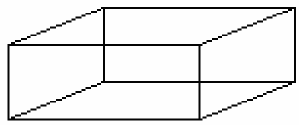

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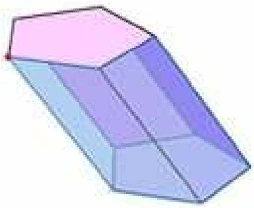

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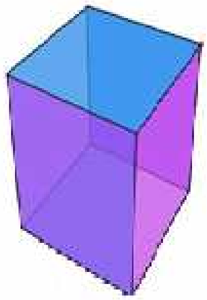
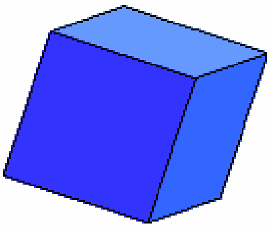
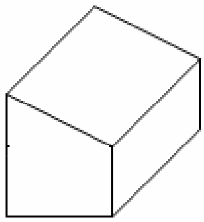


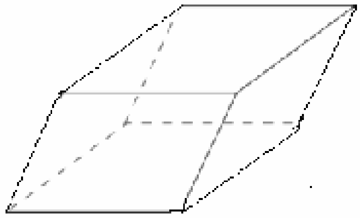
(7)



(8)



(9)

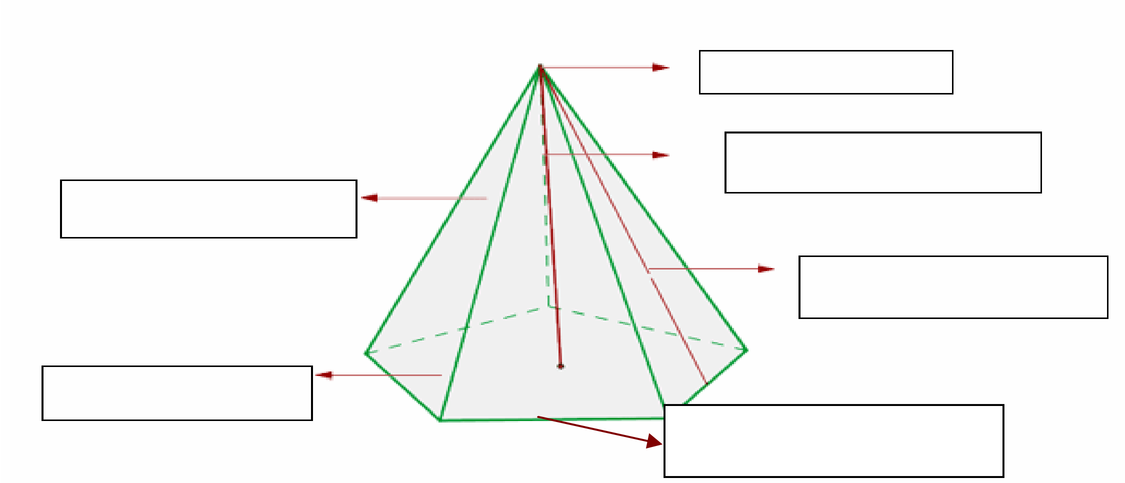


Ma Josep Sanz Espuny
IES Antoni Cumella (Granollers)

Activity 3

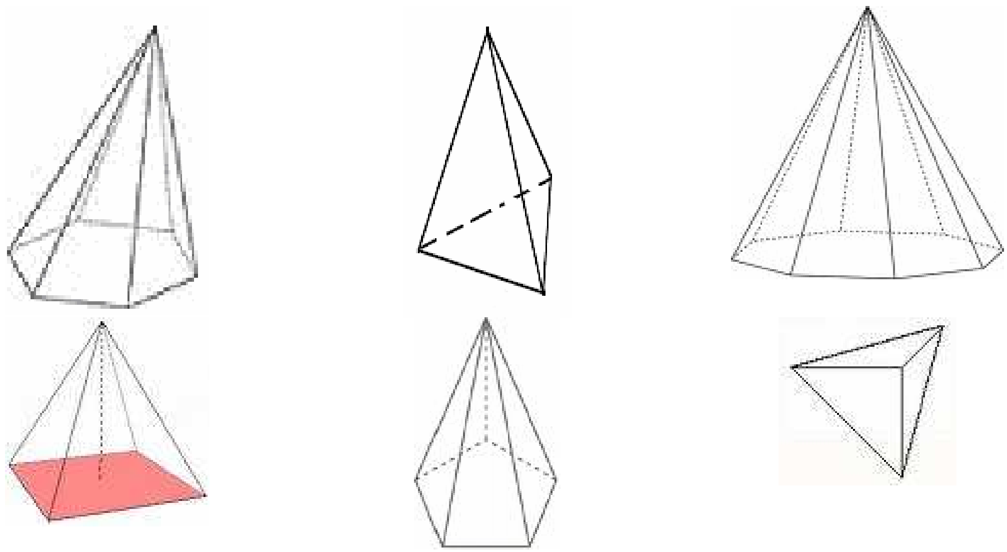
a) Read the box below and use the names in bold to label the elements of the pyramid.

A pyramid is a polyhedron formed by connecting a polygon (known as **the base**) and a point, called **the apex or the vertex**. Each **base edge** and vertex form triangles (known as the **lateral faces**).
The distance between the apex and the base is called the **height** of the pyramid.
The height of the lateral faces is called **slant height**.



b) Read the box and name the pyramids below.

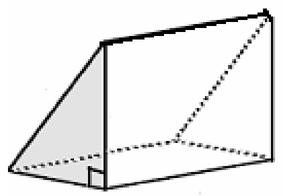
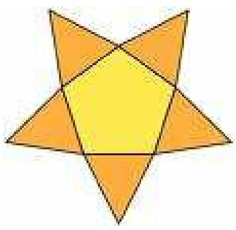
Pyramids are generally named by their bases. For example, the Egyptian pyramids have square bases, and are therefore called square pyramids.



Ma Josep Sanz Espuny
IES Antoni Cumella (Granollers)

Activity 4

Fill in the following table.

Name	Picture	Net
Cuboid		
		
		
Hexagonal prism.		

Activity 5

a) Explore the polyhedrons listed below (you can decide the last one). For each shape, determine the number of faces, edges, and vertices. Record your results below.

Polyhedron	Faces (F)	Vertices (V)	Edges (E)	F + V
Tetrahedron				
Cube				
Triangular prism				
Pentagonal pyramid				

b) Add the number of faces and the number of vertices for the shapes in the table. Compare the sum of faces and vertices to the number of edges. What did you find out? Is there a rule for all the shapes?

c) The rule you have just found is the Euler's Theorem. Express the theorem in the box.

Euler's Theorem:

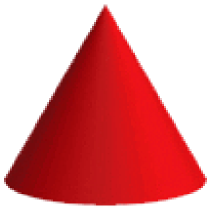
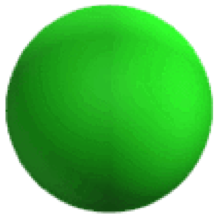
Cylinders, cones and spheres

These three types of 3D shapes are not polyhedrons. These figures have curved surfaces, not flat faces.

A **cylinder** is similar to a prism, but its two bases are circles, not polygons. Also, the sides of a cylinder are curved, not flat.

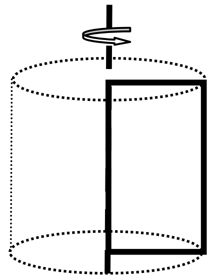
A **cone** has one circular base and a vertex that is not on the base.

The **sphere** is a space figure having all its points an equal distance from the centre point.

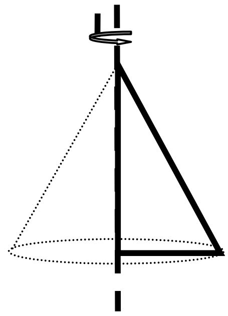
  

We call them **solids of revolution** because they are obtained by rotating a 2D shape around a straight line (called the axis of revolution)

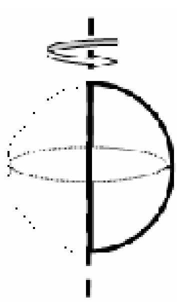
A cylinder is made by rotating a rectangle around one of its sides.



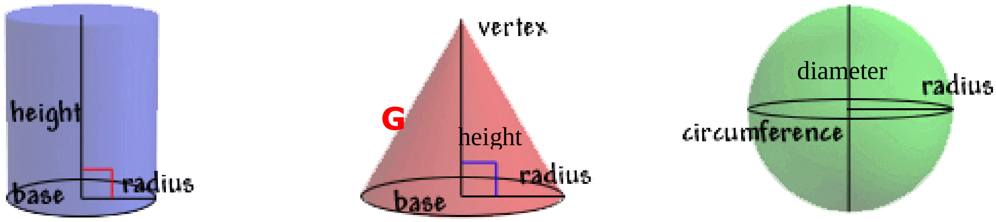
A cone is made by rotating a right angled triangle around one of its two short sides.



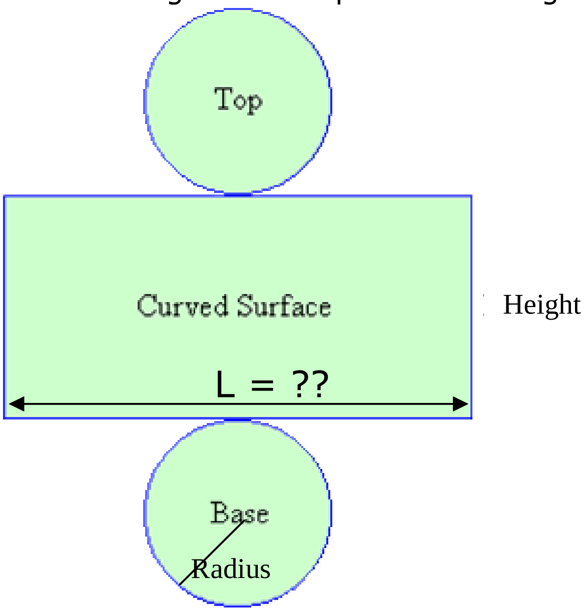
A sphere is made by rotating a semicircle around its diameter.



Have a look to its elements and its nets.



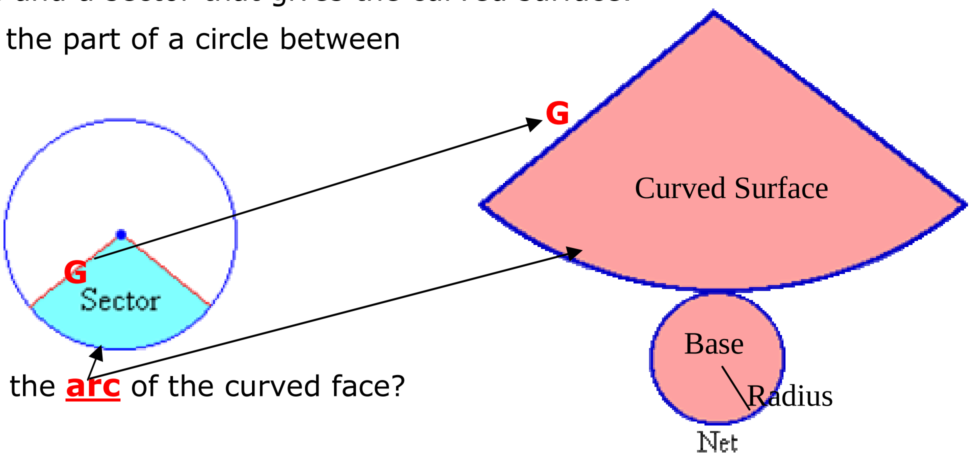
1. The net of a cylinder consists of three parts: a circle gives the base and another circle gives the top and a rectangle gives the curved surface.



• How long is the base of the rectangle?

2. The net of a cone consists of the following two parts: a circle that gives the base and a sector that gives the curved surface.

A **sector** is the part of a circle between two radii.



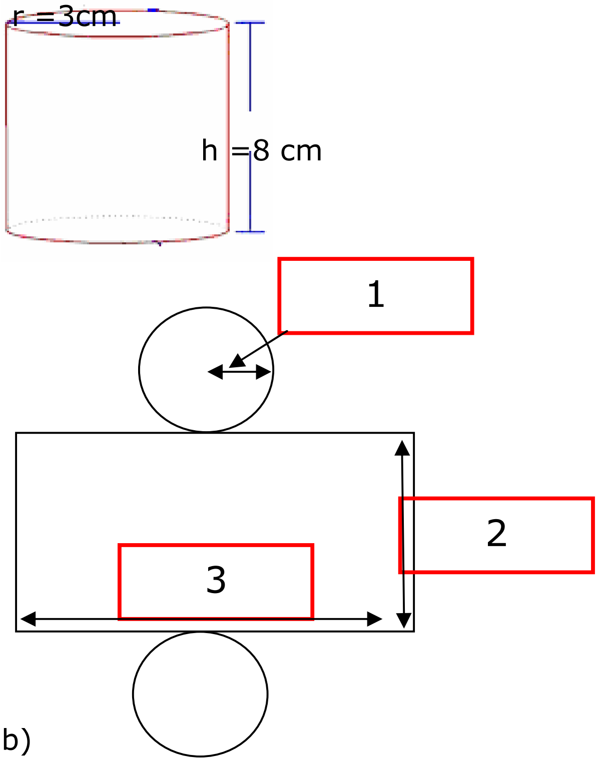
How long is the **arc** of the curved face?

3. There isn't a net for the sphere. We can't represent a sphere in two dimensions. It is not a developable surface.

Activity 1

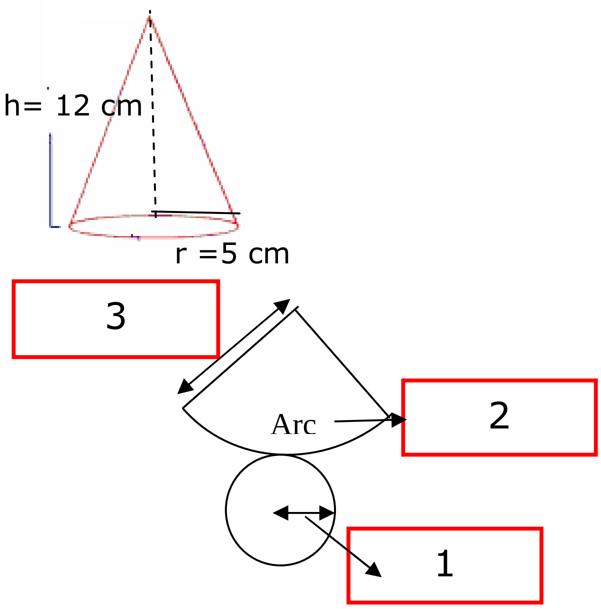
Fill in the labels in the nets using the measures in the 3D shapes.

a)



1	2	3

b)



1	2	3

Activity 2

This soup can is 12 cm tall and the radius of the bottom and top circle is 4 cm. Which are the dimensions of the paper label of the soup can?

Activity 3

To make this clown hat we have use a sector of a circle of 20cm of radius.
The arc of the sector measure 60cm. How tall is the hat?
If the clown needs a hat which radius is 7 cm, does this hat suits him?

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Lesson 4

Surface area and volume of 3D shapes

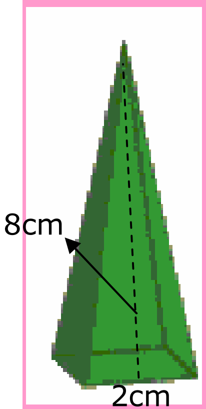
- **Worksheet 1:** Surface area
- **Worksheet 2:** Units of volume
- **Worksheet 3:** Volume of prisms and cylinders
- **Worksheet 4:** Volume of pyramids and cones

Have you ever wrapped a birthday gift?
If so, then you've covered the surface area of a polyhedron with wrapping paper.

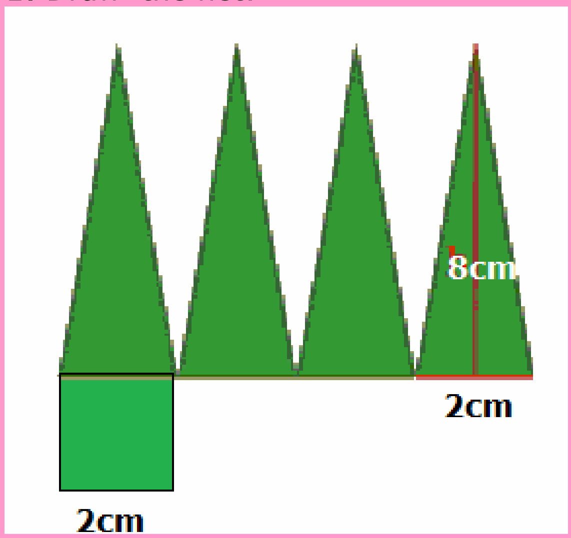
The Surface Area has two parts: the area of the lateral faces (the ***Lateral Area***) and the area of the base (the ***Base Area***).
To find the surface area of any shape, you can follow the process described:

- Draw a net of the polyhedron.
- Calculate the area of each lateral face.
- Add up the area of all the lateral faces to find the Lateral Area ($\mathbf{A_L}$).
- Calculate the area of the bases ($\mathbf{A_B}$).
- Add up both areas to find the surface area. $\mathbf{A = A_L + A_B}$

Example: To calculate the surface area of the following quadrangular pyramid we follow the steps.



1. Draw the net:



2. Calculate the area of each lateral face and add up the area of all of them.

Triangle area $= \mathbf{\frac{b \cdot h}{2} = \frac{8 \cdot 2}{2} = 8cm^2}$

$\mathbf{A_L = 4 \cdot 8 = 32cm^2}$ (8 + 8 + 8 + 8)

3. Calculate the area of the base.

$\mathbf{A_B = s^2 = 2^2 = 4cm^2}$

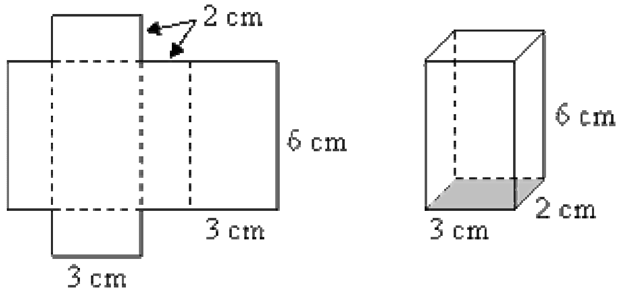
4. Add up both areas.

$\mathbf{A = A_L + A_B = 32 + 4 = 36cm^2}$

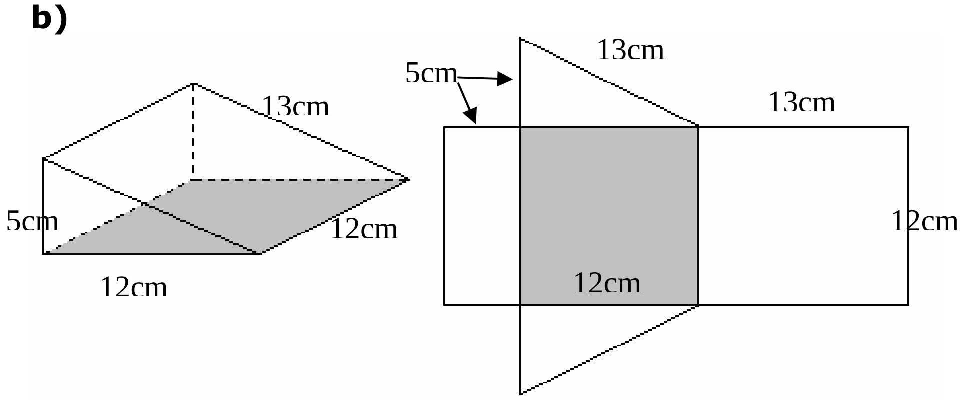
Activity 1

Find the area of these two prisms

a)



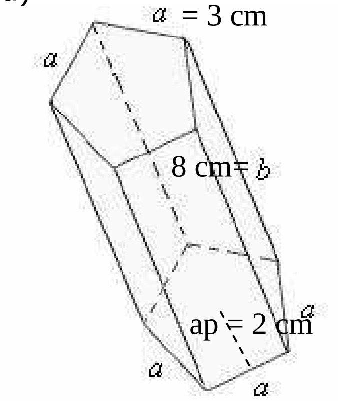
b)



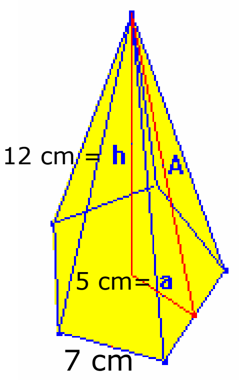
Activity 2

Calculate the surface area of the polyhedrons below.

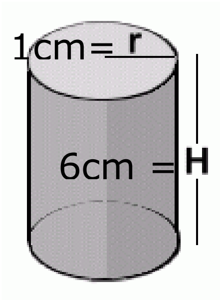
a)



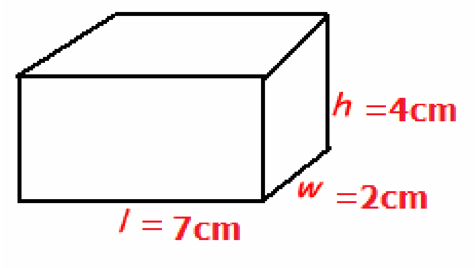
b)



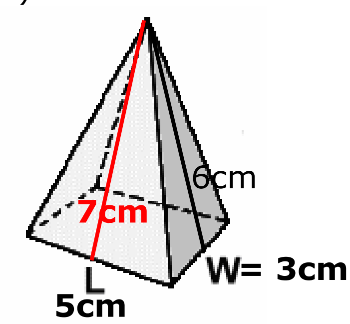
c)



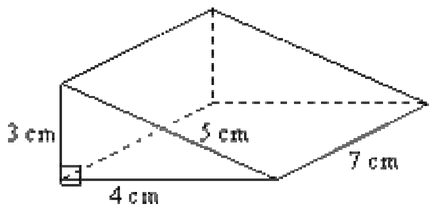
d)



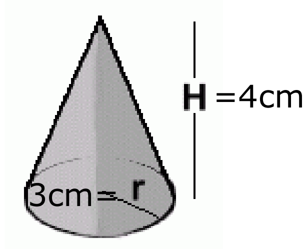
e)



f)



g)



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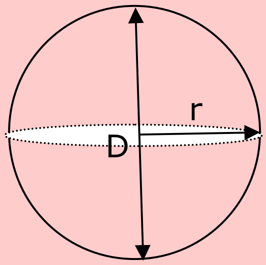
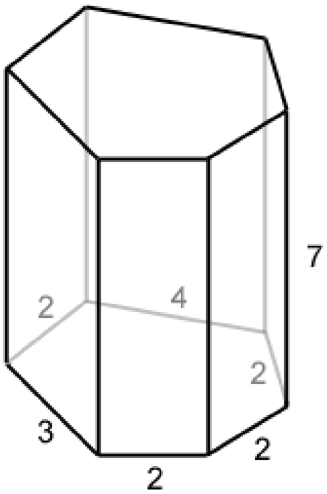
Although the **sphere** it is not a developable surface it has a surface area.
To find the surface area of the sphere we will use the following formula:

$$\mathbf{A = 4\pi r^2}$$



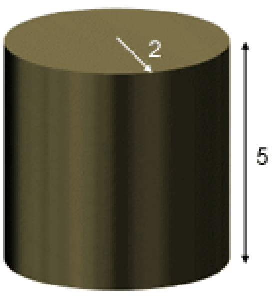
Activity 3

The height of the right prism shown in the figure is 7cm. The sides of the base polygon of are 2cm, 2cm, 2cm, 4cm, 2cm and 3cm, respectively. Calculate its lateral area.



Activity 4

What is the surface area of the cylinder shown in the figure? The radius of its base is 2 cm and its height is 5 cm.



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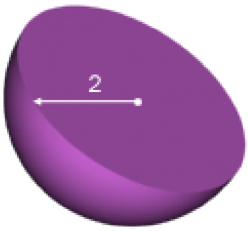
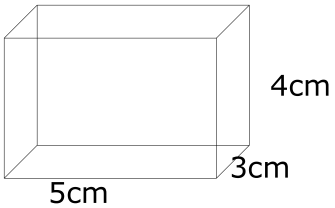
Activity 5

The hemisphere shown in the figure is produced by cutting a sphere with a plane through the center of the sphere. What is the surface area of this hemisphere (including the base circle area) if its radius is 2?



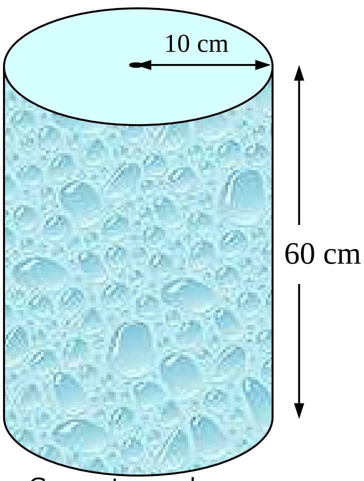
Activity 6

How many boxes as the one shown can you make with 720cm^2 of cardboard?



Activity 7

Mrs Gamp is going to cover the curved surface of a cylindrical umbrella stand with waterproof fabric. The radius is 10 cm and the height is 60 cm. Calculate the area of material required.

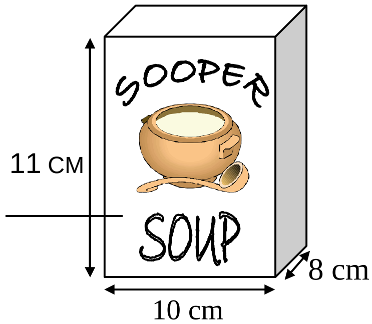


Activity 8

A cola-can has a diameter of 6.8cm and a height of 9.183cm. How much aluminium is needed for an economy pack of six cans?

Activity 9

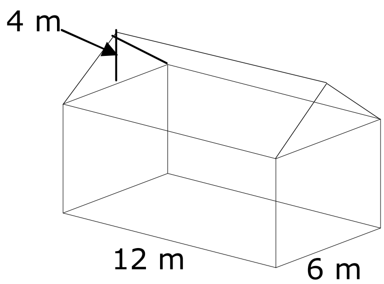
Calculate the surface area of the carton shown.



Activity 10

A man decides to paint the ceiling of his garage and to re-concrete the floor.
The paint costs £1.20 per square metre to apply and the concrete £6.80 per square metre.

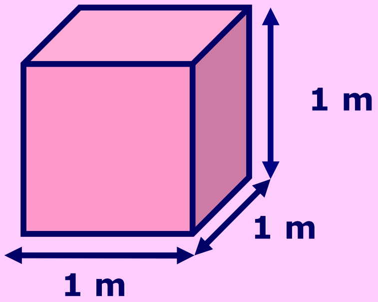
Calculate the total cost of his DIY.



Have you ever poured yourself a glass of milk? If so, then you've filled the volume of a glass with liquid.

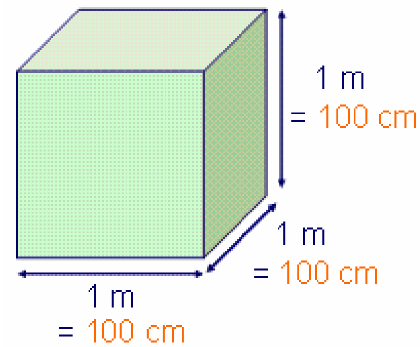
Volume is all of the space inside a three-dimensional object

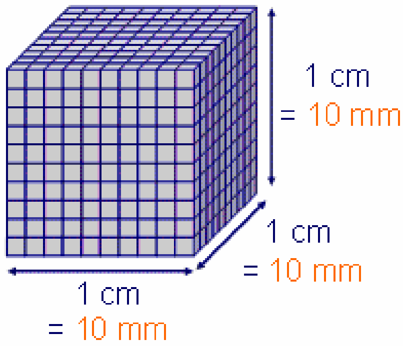
The **cubic metre** (**m³**) is the SI derived unit of volume. It is the volume of a cube with edges one metre in length.
It has multiples and submultiples.



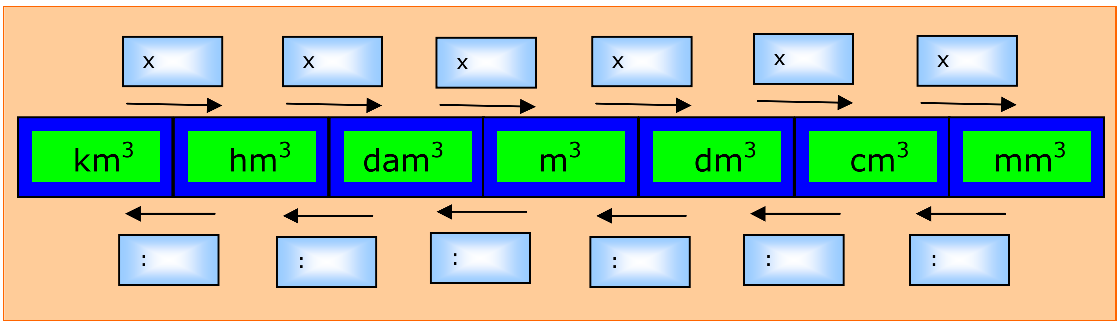
Activity 1

Look very carefully at these pictures and fill in the gaps in the conversion table.





$1m^3 =$? cm^3 $1cm^3 =$? mm^3



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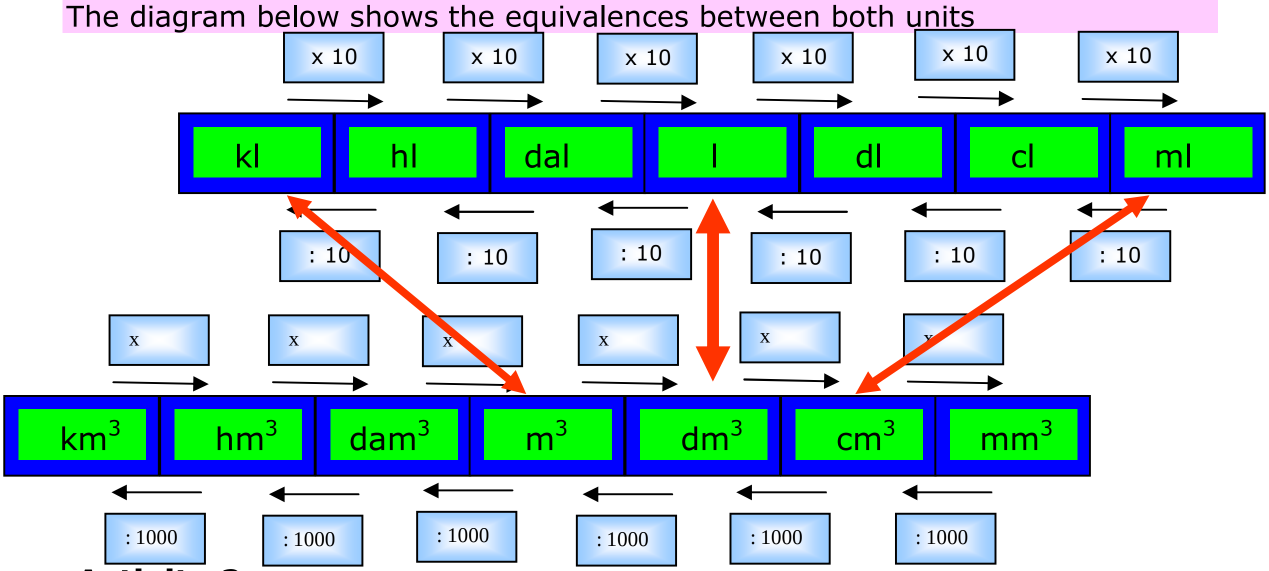
You also know other units of volume, the **units of liquid volume**. The **litre** is the main unit and its multiples and submultiples are the kilolitres, hectolitres, dekalitres, decilitres, centilitres and millilitres.

The diagram below shows the equivalences between both units



Activity 2

Convert the following measures to the indicated unit as shown in the example.

a. $2\ km^3 = 2 \cdot 1000000000 = 2000000000\ m^3$

b. $35\ mm^3 = 35 : 1000 = 0{,}035\ cm^3$

c. $135\ m^3 =$ $=$ cm^3

d. $315\ dam^3 =$ $=$ hm^3

e. $3{,}75\ dm^3 =$ $=$ mm^3

f. $31{,}235\ km^3 =$ $=$ dam^3

g. $935{,}6\ dam^3 =$ $=$ km^3

h. $35000\ m^3 =$ $=$ km^3

i. $0{,}00635\ l = 0{,}00635\ dm^3 = 0{,}00635 \cdot 1000 = 6{,}35\ cm^3$

j. $0{,}789315\ dal =$ $=$ mm^3

k. $3578{,}96\ ml =$ $=$ m^3

l. $5{,}468\ m^3 = 5{,}468 \cdot 1000\ \ 5468\ dm^3 = 5468\ l = 5468 : 10 = 546{,}8\ dl$

m. $9{,}8532\ dam^3 =$ kl

n. $67895{,}45\ mm^3 =$ l

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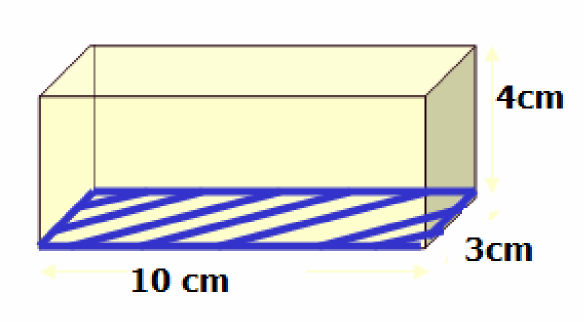
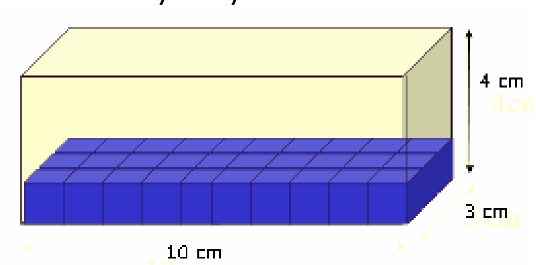
Activity 1

Read the explanation about how to find the volume of a cuboid and find a formula for the volume of all the prisms and the cylinders.

- How many 1 cm cubes will fit into this box ?



- You can start filling the base with cubes of one cubic centimetre, how many do you need?

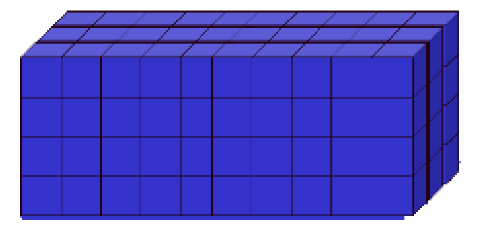


We use 10 cubes in each row and we have 3 rows.

$10 \cdot 3 = 30$ cubes in a layer

This is the area of the base of the prism

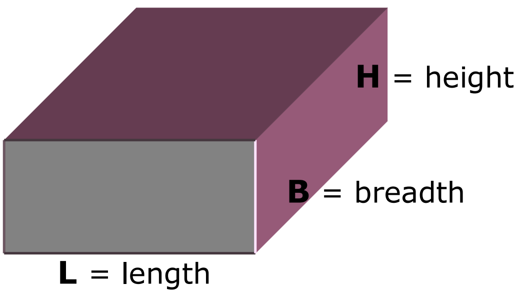
- How many layers the cuboid has?



30 cubes in a layer times 4 layers equals 120 cubes will fit into the box

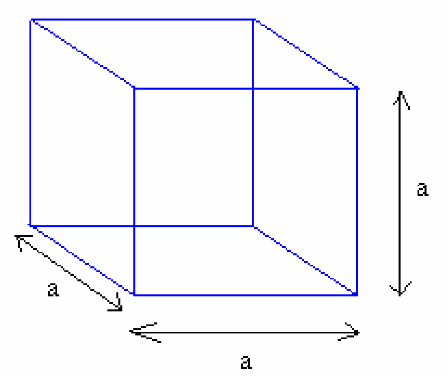
The area of the base times the height of the prism

Volume of a cuboid



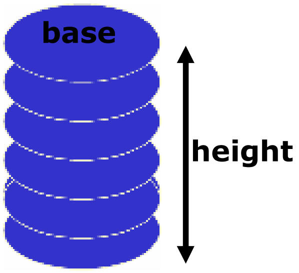
$$V = L \cdot B \cdot H$$

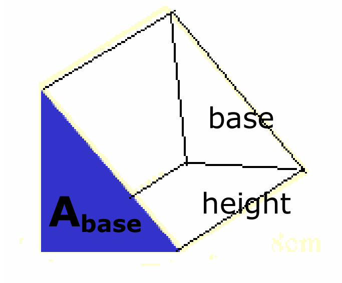
Volume of a cube

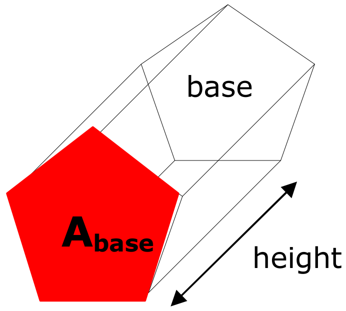


$$V = a^3$$

And now look at the pictures and find a general formula for their volume.







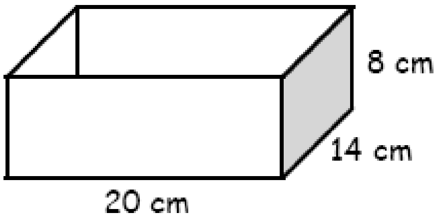
$A_{base} = \pi r^2$

$V_{cylinder} =$ $V_{prism} =$

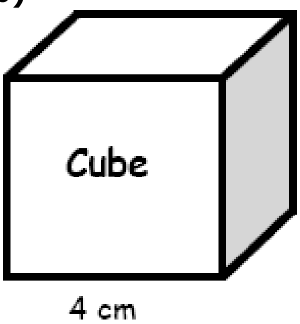
Activity 2

Calculate the volume of each (cm^3) and write how many millilitres each will hold.

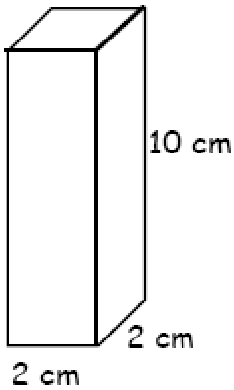
a)



b)

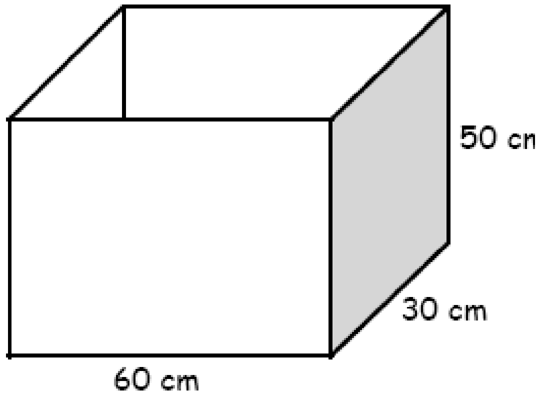


c)



Activity 3

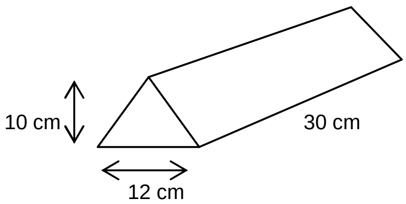
An empty aquarium has dimensions as shown.
The tank must be at least **three quarters full** of water for the fish to survive.
What is the minimum volume of water in litres that must be poured into the tank?



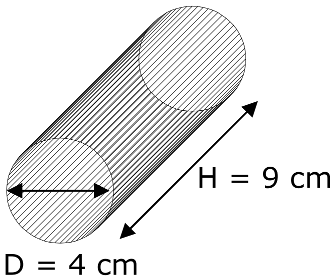
Activity 4

Calculate the volume of the shapes below

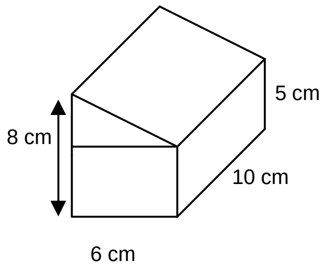
a)



b)

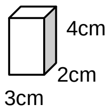


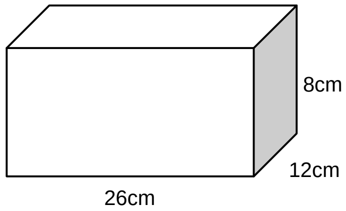
c)



Activity 5

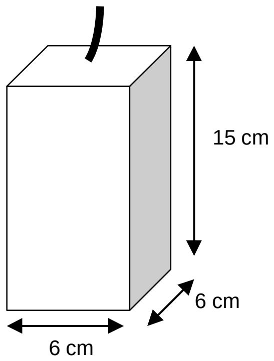
Calculate the volume of the small box on the right. How many of the small boxes below can I fit into the large box?





Activity 6

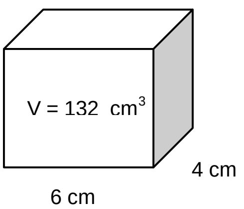
Alison has started a small business making wax candle
She makes only one size of candle and it is in the shape of a cuboid.
The base of the candle is a square of side 6 centimetres
The height of the candle is 15 centimetres
Alison buys wax in 10 litre tubs.
How many candles can she make from a tub of wax?



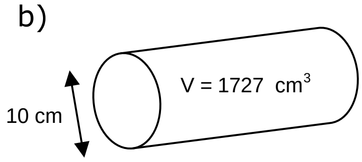
Activity 7

What is the height of the following shapes?

a)



b)



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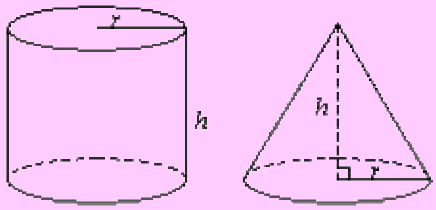
Consider the cylinder and cone shown below



The radius (r) of the bottom of the cone and the top of the cylinder are equal. The height (h) of the cone and the cylinder are equal.

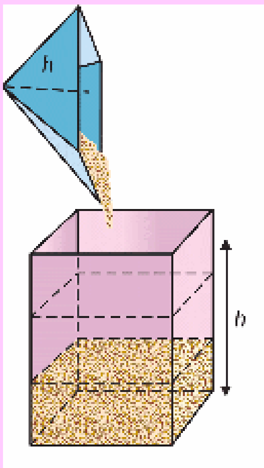
If you filled the cone with water and emptied it into the cylinder, how many times would you have to fill the cone to completely fill the cylinder to the top?

The answer is **three times**. This shows that the cylinder has three times the volume of a cone with the same height and radius.

Then the formula for the volume of a cone is :

$$\mathbf{V_{con} = \frac{1}{3}\pi r^2 h}$$

The same happens with a prism and a pyramid with the same height and equal bases.



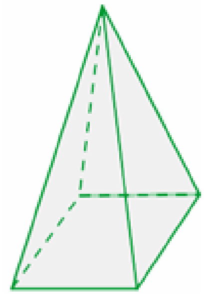
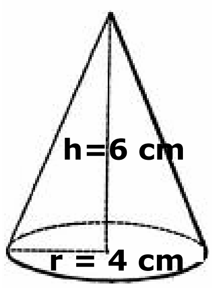
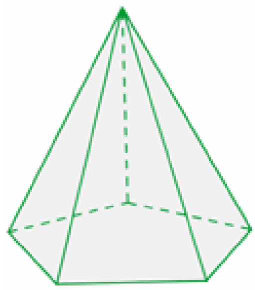
$$\mathbf{V_{pyramid} = \frac{1}{3}A_b h}$$

The formula for the volume of a sphere is:

$$\mathbf{V_{sphere} = \frac{4}{3}\pi r^3}$$

Activity 1

Calculate the volume of the shapes below

<table>
<tr>
<td>a)
</td>
<td>b)

</td>
<td>c)
</td>
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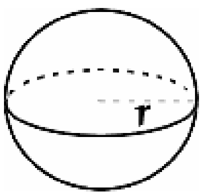
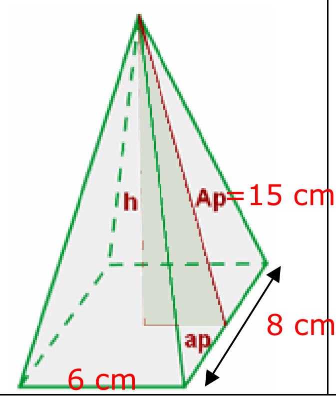
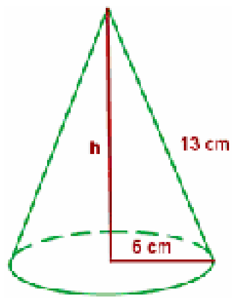
<td>e)


r = 3 cm</td>
<td>f)

</td>
<td>g)

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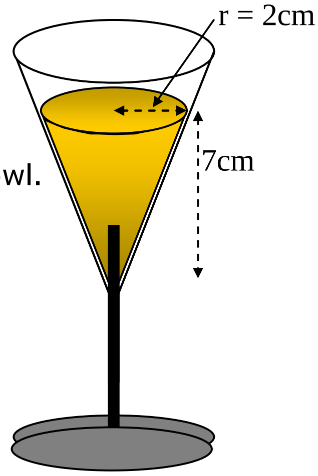
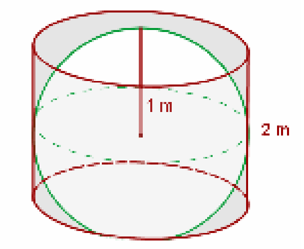
Activity 2

For a birthday party a bowl is completely filled with fruit punch and then transferred into cone shaped glasses like the one shown in the diagram below.
The depth of the liquid in each glass is 7cm
and 50 glasses can be filled from the liquid in the bowl.
Calculate the volume of the bowl



Activity 3

How many litres of water holds the space between the cylinder and the sphere?



Activity 4

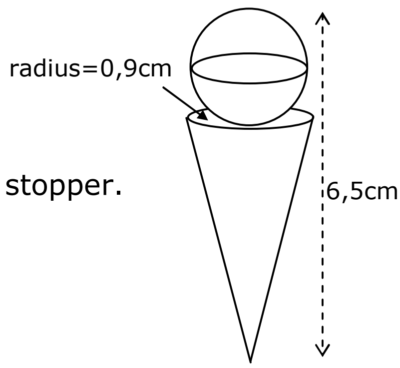
A pharmacist is filling medicine capsules. The capsules are cylinders with half spheres on each end. If the length of the cylinder is 12 mm and the radius is 2 mm, how many cubic mm of medication can one capsule hold?

Activity 5

A metal bottle stopper is made up from a cone topped with a sphere.

The sphere has diameter 1,5cm.
The cone has radius 0,·9cm
The overall length of the stopper is 6·5cm.

Calculate the volume of metal required to make the stopper.



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