Límits de successions

Nom: **Grup:**

1. Calcula els límits següents:

a) $\lim \left(\dfrac{5n^2+3n-1}{2n^2+8} - \dfrac{2n^6+2008n}{5n^6-2} \right) =$

(0,75 punts)

b) $\lim \left(5n^2 - \sqrt[5]{32n^{12} - 5n^2} \right) =$

(0,75 punts)

c) $\lim \dfrac{\dfrac{n}{n-1} - \dfrac{n-1}{n+1}}{1 + \dfrac{n-n^2}{n^2+3n}} =$

(1 punt)

d) $\lim \dfrac{2+4+6+........+2n}{n^3-1} =$

(1 punt)

e) $\lim \left(n-4-\sqrt{n^2-5n+7} \right) =$

(1 punt)

f) $\lim \left(\dfrac{5n^3+8n^2+6n}{6n^3+8n} \right)^{\frac{n^3+4}{-n^2+6}} =$

(0,75 punts)

g) $\lim \left(\dfrac{5n^2+3n-1}{5n^2+8} \right)^{\frac{7n^2+1}{n-1}} =$

(1,5 punts)

h) $\lim \sqrt[n^2]{\left(\dfrac{8n+4}{5n+3} \right)^{-n^3+4}} =$

(0,75 punts)

2. Troba a i b sabent que:

a) $\lim \dfrac{an^3+2n^2-n+1}{bn^4-2n^3+5} = \dfrac{37}{2}$

(1,25 punt)

b) $lim \left(\dfrac{1-an^2}{3n^2-2} \right)^{1-bn^2} = \sqrt{e}$

(1,25 punts)

Solució

Nom: **Grup:**

1. Calcula els límits següents:

a) $\lim\left(\frac{5n^2+3n-1}{2n^2+8}-\frac{2n^6+2008n}{5n^6-2}\right)=\frac{5}{2}-\frac{2}{5}=\frac{25-4}{10}=\frac{21}{10}$

(0,75 punts)

b) $\lim\left(5n^2-\sqrt[5]{32n^{12}-5n^2}\right)=$

$$=\lim\left(5n^2-\sqrt[5]{32n^{12}}\right)=\lim\left(5n^2-2n^{12/5}\right)=\lim\left(-2n^{12/5}\right)=-\infty$$

(0,75 punts)

c) $\lim\frac{\frac{n}{n-1}-\frac{n-1}{n+1}}{1+\frac{n-n^2}{n^2+3n}}=\lim\frac{\frac{n^2+n-n^2+2n-1}{(n-1)(n+1)}}{\frac{n^2+3n+n-n^2}{n^2+3n}}=\lim\frac{\frac{3n}{n^2}}{\frac{4n}{n^2}}=\lim\frac{3n}{4n}=\frac{3}{4}$

(1 punt)

d) $\lim\frac{2+4+6+\dots\dots\dots+2n}{n^3-1}=$

$$\lim\frac{\frac{(2+2n)n}{2}}{n^3-1}=\lim\frac{2n^2}{2}\cdot\frac{1}{n^3}=\lim\frac{1}{n}=0^+$$

(1 punt)

e) $\lim\left(n-4-\sqrt{n^2-5n+7}\right)=$

$$\lim\left(n-4-\sqrt{n^2-5n+7}\right)\frac{\left(n-4+\sqrt{n^2-5n+7}\right)}{\left(n-4+\sqrt{n^2-5n+7}\right)}=\lim\frac{(n-4)^2-\left(n^2-5n+7\right)}{\left(n-4+\sqrt{n^2-5n+7}\right)}=$$

$$\lim\frac{n^2-8n+16-n^2+5n-7}{\left(n+\sqrt{n^2}\right)}=\lim\frac{-3n}{2n}=\frac{-3}{2}$$

(1 punt)

f) $\lim\left(\frac{5n^3+8n^2+6n}{6n^3+8n}\right)^{\frac{n^3+4}{-n^2+6}}=\left(\frac{5}{6}\right)^{-\infty}=\left(\frac{6}{5}\right)^{+\infty}=+\infty$

(0,75 punts)

g) $\lim\left(\frac{5n^2+3n-1}{5n^2+8}\right)^{\frac{7n^2+1}{n-1}}=$

és una indeterminació del tipus 1^∞ així doncs anem a fabricar el límit del nombre e

$$\left(1+\frac{5n^2+3n-1}{5n^2+8}-1\right)^{\frac{7n^2+1}{n-1}}=\left(1+\frac{5n^2+3n-1-5n^2-8}{5n^2+8}\right)^{\frac{7n^2+1}{n-1}}=\left(1+\frac{3n-9}{5n^2+8}\right)^{\frac{7n^2+1}{n-1}}=$$

$$= \left(1+\frac{1}{\frac{5n^2+8}{3n-9}}\right)^{\frac{7n^2+1}{n-1}} = \left(1+\frac{1}{\frac{5n^2+8}{3n-9}}\right)^{\frac{5n^2+8}{3n-9}\cdot\frac{7n^2+1}{n-1}\cdot\frac{3n-9}{5n^2+8}} = \left[\left(1+\frac{1}{\frac{5n^2+8}{3n-9}}\right)^{\frac{5n^2+8}{3n-9}}\right]^{\frac{7n^2+1}{n-1}\cdot\frac{3n-9}{5n^2+8}} \to$$

$$\to e^{\frac{21}{5}} = \sqrt[5]{e^{21}} = e^4\cdot\sqrt[5]{e}$$

(1,5 punts)

h) $\text{lím}\ \sqrt[n^2]{\left(\frac{8n+4}{5n+3}\right)^{-n^3+4}} = \text{lím}\ \left(\frac{8}{5}\right)^{\frac{-n^3}{n^2}} = \left(\frac{8}{5}\right)^{-\infty} = \left(\frac{5}{8}\right)^{+\infty} = 0^+$

(0,75 punts)

2. Troba a i b sabent que:

a) $\text{lím}\ \frac{an^3+2n^2-n+1}{bn^4-2n^3+5} = \frac{37}{2}$

Per tal que el grau del numerador i denominador sigui el mateix cal que b=0

i aleshores després fent el límit surt que $\frac{a}{-2} = \frac{37}{2}$ amb la qual cosa a= –37.

Així doncs la solució és b=0 i a=–37

(1,25 punt)

b) $lim \left(\frac{1-an^2}{3n^2-2}\right)^{1-bn^2} = \sqrt{e}$

Per tal que la base tingui límit 1 cal que a=–3 i aleshores després calculant el límit d'indeterminació 1^∞ resulta que:

$$\left(1+\frac{1+3n^2}{3n^2-2}-1\right)^{1-bn^2} = \left(1+\frac{1-3n^2+3n^2+2}{3n^2-2}\right)^{1-bn^2} = \left(1+\frac{3}{3n^2-2}\right)^{1-bn^2} = \left(1+\frac{1}{\frac{3n^2-2}{3}}\right)^{1-bn^2} =$$

$$= \left(1+\frac{1}{\frac{3n^2-2}{3}}\right)^{\frac{3n^2-2}{3}\cdot(1-bn^2)\cdot\frac{3}{3n^2-2}} = \left[\left(1+\frac{1}{\frac{3n^2-2}{3}}\right)^{\frac{3n^2-2}{3}}\right]^{\frac{3(1-bn^2)}{3n^2-2}} \to e^{\frac{-3b}{3}} = e^{-b}$$

Així doncs com

$$e^{-b} = e^{1/2} \Rightarrow b = \frac{-1}{2}$$

Solució a=–3 i b=–1/2

(1,25 punts)