Exercici 1

a) $z_1 - 2z_1 \cdot z_2 = (1-i) - 2\cdot(1-i)\cdot(5+2i) = -13+5i$.

b) Passem a forma polar $z_1 = 1 - i$. Com $|z_1| = \sqrt{1^2 + (-1)^2} = \sqrt{2}$ i φ = arctag(-1) = 315º, tenim que $z_1 = \sqrt{2}_{315^\circ}$.

$(z_1)^8 = (\sqrt{2}_{315^\circ})^8 = (\sqrt{2})^8{}_{8\cdot 315^\circ} = 16_{2520^\circ} = 16_{0^\circ} = 16\cdot(\cos 0^\circ + i\cdot\sin 0^\circ) = 16\cdot(1+0i) = 16$.

Exercici 2

a) Com el conjugat de $z = 3 - \sqrt{3}i$ és $\bar{z} = 3 + \sqrt{3}\cdot i$, l'oposat és $-z = -3 + \sqrt{3}\cdot i$, i l'oposat del conjugat de z és $-\bar{z} = -3 - \sqrt{3}\cdot i$, tenim que les dimensions del rectangle han de ser: 6 i $2\cdot\sqrt{3}$, per tant: $A = 12\cdot\sqrt{3}\ u^2$.

b) Les solucions de l'equació $x^2 - 2x + 10 = 0$ són:

$x = \dfrac{2 \pm \sqrt{4-40}}{2} = \dfrac{2 \pm 6i}{2} = 1 \pm 3i$, és a dir $x_1 = 1 + 3i$ i $x_2 = 1 - 3i$.

$x_1 + x_2 = (1+3i) + (1-3i) = 2$.

$x_1 \cdot x_2 = (1+3i)\cdot(1-3i) = 1^2 - (3i)^2 = 1 - 9i^2 = 1 + 9 = 10$.

Exercici 3

Passem a forma polar $z = -8 + 8\sqrt{3}i$. Tenim:

$|z| = \sqrt{(-8)^2 + (8\sqrt{3})^2} = \sqrt{256} = 16$ i $\varphi = \text{arctag}(-\sqrt{3}) = 120^\circ$.

$$\sqrt[4]{-8+8\sqrt{3}i} = \sqrt[4]{16_{120^\circ}} = 2_{\frac{120^\circ + 360^\circ\cdot k}{4}} = \begin{cases} 2_{30^\circ} = 2\cdot(\cos 30^\circ + i\cdot\sin 30^\circ) = \sqrt{3} + i \\ 2_{120^\circ} = 2\cdot(\cos 120^\circ + i\cdot\sin 120^\circ) = -1 + \sqrt{3}\cdot i \\ 2_{210^\circ} = 2\cdot(\cos 210 + i\cdot\sin 210^\circ) = -\sqrt{3} - i \\ 2_{300^\circ} = 2\cdot(\cos 300^\circ + i\cdot\sin 300^\circ) = 1 - \sqrt{3}\cdot i \end{cases}$$

Exercici 4

Com $\dfrac{1+mi}{2-i} = \dfrac{(1+mi)\cdot(2+i)}{(2-i)\cdot(2+i)} = \dfrac{2-m}{5} + \dfrac{1+2m}{5}\cdot i$, tenim que:

a) si ha de ser un imaginari pur, $\dfrac{2-m}{5} = 0$, d'on m = 2.

c) Si ha de ser un nombre real, $\dfrac{1+2m}{5} = 0$, d'on $m = -\dfrac{1}{2}$.

Exercici 5

Si $z = a + bi$, alehores $\bar{z} = a - bi$, i per tant, tenim: $\dfrac{z-\bar{z}}{z+\bar{z}} = \dfrac{(a+bi)-(a-bi)}{(a+bi)+(a-bi)} = \dfrac{2bi}{2a} = \dfrac{b}{a}\cdot i$.

Celestí Bertran i Infante
IES l'Alzina