

SOLUCIÓ DE L'EXÀMEN PARCIAL DE CÀLCUL DE PRIMITIVES

Exercici 1:

$$\begin{aligned}\int \frac{x^2}{\sqrt{1-x^3}} dx &= \int x^2 \cdot (1-x^3)^{-1/2} dx = -\frac{1}{3} \int 3x^2 \cdot (1-x^3)^{-1/2} dx = -\frac{1}{3} \cdot \frac{(1-x^3)^{1/2}}{1/2} + C = \\ &= -\frac{2}{3} \sqrt{1-x^3} + C\end{aligned}$$

Exercici 2:

$$\int \frac{e^{\operatorname{arctag} x}}{1+x^2} dx = \int e^{\operatorname{arctag} x} \cdot \frac{1}{1+x^2} dx = e^{\operatorname{arctag} x} + C$$

Exercici 3:

$$\int \frac{5 \cdot \ln x}{x} dx = 5 \cdot \int \ln x \cdot \frac{1}{x} dx = 5 \cdot \frac{(\ln x)^2}{2} + C$$

Exercici 4:

$$\int \cos(1+3e^x) \cdot e^x dx = \frac{1}{3} \int \cos(1+3e^x) \cdot 3e^x dx = \frac{1}{3} \cdot \sin(1+3e^x) + C$$

Exercici 5:

$$\int \frac{1-2e^x}{e^x} dx = \int (e^{-x} - 2) dx = -e^{-x} - 2x + C$$

Exercici 6:

$$\begin{aligned}\int \sin^2 x \cdot \cos^3 x dx &= \int \sin^2 x \cdot (1 - \sin^2 x) \cdot \cos x dx = \int \sin^2 x \cdot \cos x dx - \int \sin^4 x \cdot \cos x dx = \\ &= \frac{\sin^3 x}{3} - \frac{\sin^5 x}{5} + C\end{aligned}$$

Exercici 7:

Per integració per parts, tenim:

$$f(x) = x + 1 \rightarrow f'(x) = 1$$

$$g'(x) = e^{-x} \rightarrow g(x) = -e^{-x}$$

$$\int (x+1) \cdot e^{-x} dx = -(x+1) \cdot e^{-x} - \int (-1) \cdot e^{-x} dx = -(x+1) \cdot e^{-x} - e^{-x} + C$$

Exercici 8:

Resolent l'equació $x^3 + 5x^2 + 6x = 0$, obtenim tres solucions reals simples: $x = 0$, $x = -2$ i $x = -3$, per tant, podem escriure:

$$\frac{2x-3}{x^3+5x^2+6x} = \frac{A}{x} + \frac{B}{x+2} + \frac{C}{x+3} = \frac{A \cdot (x+2) \cdot (x+3) + B \cdot x \cdot (x+3) + C \cdot x \cdot (x+2)}{x \cdot (x+2) \cdot (x+3)}, \text{ d'on tenim:}$$

$$2x - 3 = A \cdot (x+2) \cdot (x+3) + B \cdot x \cdot (x+3) + C \cdot x \cdot (x+2)$$

Si fem $x = 0$, tenim $-3 = 6A$, d'on $A = -1/2$, si fem $x = -2$, tenim $-7 = -2B$, d'on $B = 7/2$ i

si fem $x = -3$, tenim que: $-9 = 3C$, d'on $C = -3$. D'aquí tenim que:

$$\int \frac{2x-3}{x^3+5x^2+6x} dx = \int \frac{-1/2}{x} dx + \int \frac{7/2}{x+2} dx + \int \frac{-3}{x+3} dx = -\frac{1}{2} \ln|x| + \frac{7}{2} \ln|x+2| - 3 \ln|x+3| + C$$