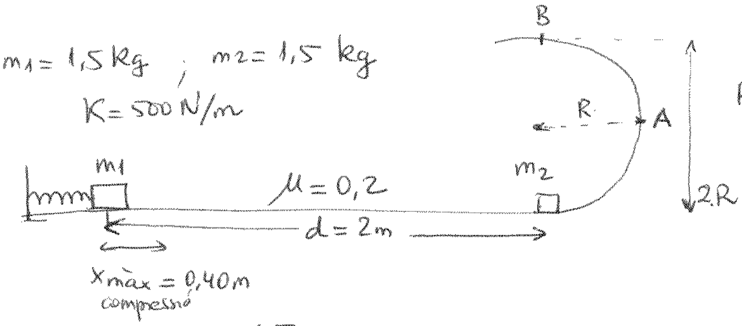
PROVA de SUFICIÈNCIA FÍSICA CORRECCIÓ FINAL (1)

*(1) $m_1 = 1{,}5\ kg$; $m_2 = 1{,}5\ kg$

$K = 500\ N/m$

$R = 0{,}25\,m$



a) $W_{nc} = W_{FF} = \Delta E_m$

(I) $\boxed{W_{FF} = -\mu\, m_1 . g . d}$ $\quad \Delta E_m = \underset{\substack{\text{abans}\\ \text{xoc}\\ (Ec_2)}}{Em_2} - \underset{\substack{\text{molla}\\ \text{comprimida abans de deixar-la}\\ (E_{P\,elàs\,1})\ \text{en llibertat}\\ v_0 = 0}}{Em_1}$

$\Delta E_m = Ec_2 - Ep_1$

(II) $\boxed{\Delta E_m = \frac{1}{2} m_1 v_1^2 - \frac{1}{2} k x_{màx}^2}$

$-0{,}2 . 1{,}5 . 9{,}8 . 2 = \frac{1}{2} . 1{,}5 . v_1^2 - \frac{1}{2} . 500 (0{,}40)^2 \Rightarrow \boxed{v_1 = 6{,}7\ m/s}$

b) $\vec{p} = \vec{ct}$ xoc elàstic

$\begin{cases} (A)\ m_1 v_1 + m_2 . \cancel{v_2} = m_1 v_1' + m_2 . v_2' \\ (B)\ v_1 + v_1' = \cancel{v_2} + v_2' \end{cases}$ (horitzontal unidimensional)

(A) $\cancel{m_1} . 6{,}7 = \cancel{m_1} . v_1' + \cancel{m_2} . v_2'$ $\quad (m_1 = m_2)$

(B) $6{,}7 + v_1' = 0 + v_2'$

$\Rightarrow \boxed{v_1' = 0\ m/s}$ $\quad \boxed{v_2' = +6{,}7\ m/s}$

c) $a_{cB} = \frac{v_B^2}{R} = \frac{v_2'^2 - 4gR}{R} = \frac{(6{,}7)^2 - 4.9{,}8.0{,}25}{0{,}25} = \boxed{140{,}36\ m/s^2}$

Per energies: $Em = ct$ (comparo just després del xoc i B)

$\boxed{\frac{1}{2} \cancel{m_2} . v_2'^2 = \cancel{m_2} . g . 2R + \frac{1}{2} \cancel{m_2} v_B^2} \Rightarrow \boxed{v_B = \sqrt{v_2'^2 - 4gR} = 5{,}9\ m/s}$

*(2) $\varphi = t^3 + 4t - 2$ (rad) ; $r = 2m$

$\omega = \frac{d\varphi}{dt} = 3t^2 + 4$; $\boxed{\omega(1) = 3.1^2 + 4 = 7\ rad/s}$

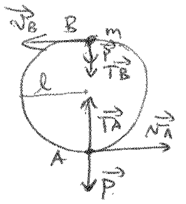
$v = \omega . r \Rightarrow \boxed{v(1) = 7 . 2 = 14\ m/s} \Rightarrow \boxed{a_n = \frac{v^2}{r} = \omega^2 . r = 98\ \frac{m}{s^2}}$

$\alpha = \frac{d\omega}{dt} = 6t \Rightarrow \boxed{\alpha(1) = 6.1 = 6\ rad/s^2}$

$\boxed{a_t(1) = \alpha . r = 6 . 2 = 12\ m/s^2}$ $\quad \boxed{a_t = \sqrt{a_n^2 + a_t^2} = \sqrt{98^2 + 12^2} = 98{,}73\ \frac{m}{s^2}}$

*(3) c) CORRECTA (examen final meu 2003/04 → web)

"La tensió en el punt A excedeix en "6mg" la tensió en el punt B"



$$\left.\begin{aligned} T_A - P &= F_{CA} = m\frac{v_A^2}{l} \\ T_B + P &= F_{CB} = m\frac{v_B^2}{l} \end{aligned}\right\} \text{Dinàmica de ROTACIÓ (I)}$$

Energies: $E_{mA} = E_{mB}$; $\frac{1}{2}\cancel{m}v_A^2 = \underbrace{\cancel{m}\cdot g\cdot 2l}_{E_{pB}} + \underbrace{\frac{1}{2}\cancel{m}v_B^2}_{E_{cB}}$

$$\boxed{v_A^2 = 4\cdot g\cdot l + v_B^2} \quad \text{(II)}$$

Per (I) si restem: $T_A - T_B = m\left(\frac{v_A^2}{l} - \frac{v_B^2}{l}\right) + 2mg$

per (II) : $T_A - T_B = m\left(\frac{(4gl + v_B^2)}{l} - \frac{v_B^2}{l}\right) + 2mg$

$$T_A - T_B = m\left(\frac{4g\cancel{l}}{\cancel{l}} + \cancel{\frac{v_B^2}{l}} - \cancel{\frac{v_B^2}{l}}\right) + 2mg = 4mg + 2mg$$

$$\boxed{T_A - T_B = 6\,mg}$$

*(4) $y(t,x) = 0{,}4\sin\pi\left(\frac{t}{2} - \frac{x}{4}\right)$ (SI).

a) v? ; λ? ; T? $\boxed{y = A\sin(\omega t - Kx) = 0{,}4\sin\left(\frac{\pi t}{2} - \frac{\pi}{4}x\right)}$

$\boxed{\omega = \frac{\pi}{2}\,s^{-1} = \frac{2\pi}{T}}$; $\boxed{T = \frac{2\pi}{\omega} = \frac{2\pi}{\frac{\pi}{2}} = 4\,s}$ $\quad \lambda = v\cdot T$

$\boxed{k = \frac{2\pi}{\lambda} = \frac{\pi}{4}\,m^{-1}}$ $\boxed{\lambda = \frac{2\pi}{K} = \frac{2\pi}{\frac{\pi}{4}} = 8\,m}$ $\quad \boxed{v = \frac{\lambda}{T} = \frac{8}{4} = 2\,m/s}$

b) $A = 0{,}4\,m$; $v_y = \frac{dy}{dt} = A\omega\cos(\omega t - kx) = 0{,}4\cdot\frac{\pi}{2}\cos\left(\frac{\pi}{2}t - \frac{\pi}{4}x\right)$

$\boxed{a_{y\,max} = \mp A\omega^2}$ $\quad a_y = \frac{d^2y}{dt^2} = -A\omega^2\sin(\omega t - kx) = -0{,}4\left(\frac{\pi}{2}\right)^2\cdot\sin\left(\frac{\pi}{2}t - \frac{\pi}{4}x\right)$

$$\boxed{a_{màx} = \mp 0{,}4\left(\frac{\pi}{2}\right)^2 = \frac{0{,}4\cdot\pi^2}{4} = \mp 0{,}1\,\pi^2\,\frac{m}{s^2} \simeq \mp 0{,}99\,m/s^2}$$

c) $(x=0\ ;\ t=6s)$

$$\boxed{y = 0{,}4 \sin\left(\frac{\pi}{2}t - \frac{\pi}{4}x\right) = 0{,}4 \cdot \sin\left(\frac{\pi}{2}\cdot 6 - 0\right) = 0{,}4 \sin 3\pi = 0\,m}$$

↓ elongació

(3π = 1 volta i mitja)

$$v_y = v = A\omega \cos(\omega t - Kx) = 0{,}4\cdot\frac{\pi}{2}\cos\left(\frac{\pi}{2}t - \frac{\pi}{4}x\right)$$

$$v(t=6\ ;\ x=0) = 0{,}2\pi \cdot \cos\left(\frac{\pi}{2}\cdot 6 - 0\right) = 0{,}2\pi \cdot \underbrace{\cos 3\pi}_{-1} = -0{,}2\pi\ m/s$$

$\boxed{v = -0{,}63\ m/s}$ → velocitat transversal

* (5) $\lambda = 5\cdot 10^{-7} m$; $W_0 = 3{,}2\times 10^{-19}$ J

a) $W_0 = h\cdot f_0$; $\boxed{f_0 = \frac{W_0}{h} = \frac{3{,}2\times 10^{-19}}{6{,}62\times 10^{-34}} = 4{,}8\times 10^{14}\ Hz\ (s^{-1})}$

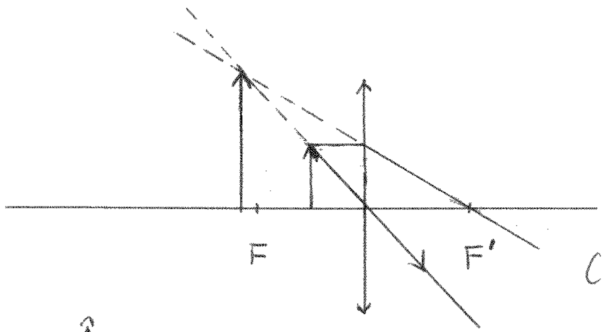
b) $E\,fotó = \underbrace{W_0}_{\text{treball d'extracció}} + Ec\,màx$

$$Ec\,màx = E\,fotó - W_0 = h\cdot f - W_0 = h\cdot\frac{c}{\lambda} - W_0$$

$f = \frac{c}{\lambda}$

$$\boxed{Ec\,màx = 6{,}62\cdot 10^{-34}\cdot\frac{3\cdot 10^{8}}{5\cdot 10^{-7}} - 3{,}2\cdot 10^{-19} = 7{,}72\times 10^{-20}\ J}$$

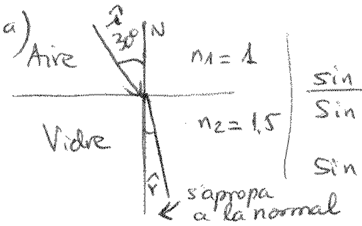
* (6)



- convergent
- imatge virtual
- imatge dreta
- ampliada

(La lent actua com una lupa)

* (7) a)



$n_1 = 1$

$n_2 = 1{,}5$

$$\frac{\sin\hat{i}}{\sin\hat{r}} = \frac{n_2}{n_1} = \frac{n_{vidre}}{n_{aire}} \Rightarrow \sin\hat{r} = \frac{n_1}{n_2}\sin\hat{i}$$

$$\sin\hat{r} = \frac{1}{1{,}5}\cdot\sin 30° = 0{,}33 \rightarrow \boxed{\hat{r} = 19{,}47°}$$

7) b) $\lambda = 7{,}6 \times 10^{-9}\ m$; $n_1 = 1$; $n_2 = 1{,}5$

- $\boxed{f = ct}$ (Només depèn de la font emissora)
- $\boxed{E = ct}$ $(E = h \cdot f)$ h: ct de Planck
- λ i v canvien i són directament proporcionals

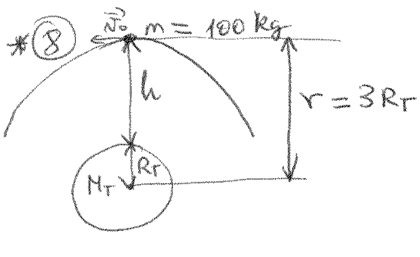
$$f_1 = f_2 \iff \boxed{\frac{v_1}{\lambda_1} = \frac{v_2}{\lambda_2}}\ ;^{(I)}$$

$$n = \frac{c}{v} \Rightarrow \boxed{v_1 = \frac{c}{n_1} = \frac{3 \cdot 10^8}{1} = 3 \cdot 10^8\ m/s} \text{ (aire)}$$

$$\Rightarrow \boxed{v_2 = \frac{c}{n_2} = \frac{3 \cdot 10^8}{1{,}5} = 2 \cdot 10^8\ m/s} \text{ (vidre).}$$

$$\Rightarrow^{(I)} \boxed{\lambda_2 = \frac{v_2}{v_1} \cdot \lambda_1 = \frac{n_1}{n_2} \cdot \lambda_1 = \frac{1}{1{,}5} \cdot 7{,}6 \times 10^{-7} = 5{,}1 \times 10^{-7}\ m}$$

*8



a) $|\vec{F_c}| = |\vec{F_G}|$

$$\cancel{m} \cdot w^2 \cdot r = \frac{G M_T \cancel{m}}{r^2} \ ; \quad w^2 = \frac{G M_T}{r^3}$$

$$w = \sqrt{\frac{G M_T}{r^3}} = \sqrt{\frac{g_0 \cdot R_T^2}{(3 \cdot R_T)^3}} = \sqrt{\frac{g_0}{27 \cdot R_T}}$$

$$\boxed{w = \sqrt{\frac{9{,}8}{27 \cdot 6{,}4 \cdot 10^6}} = 2{,}38 \times 10^{-4}\ rad/s}$$

b) $$|\vec{g}| = \frac{GM}{r^2} = \frac{g_0 \cdot R_T^2}{(3 \cdot R_T)^2} = \frac{g_0 \cdot \cancel{R_T^2}}{9 \cdot \cancel{R_T^2}} = \frac{9{,}8}{9} = 1{,}1\ m/s^2$$

- bé: $g_0 = \frac{GM}{R_T^2}$; $g_1 = \frac{GM}{r^2} = \frac{GM}{(3R_T)^2} = \frac{GM}{9R_T^2}$

$$\frac{g_0}{g_1} = \frac{r^2}{R_T^2} = \frac{(3R_T)^2}{R_T^2} = 9 \ ; \quad g_1 = \frac{g_0}{9} = 1{,}1\ m/s^2$$

c) $$E_{m\,\text{enllaç}} = +\frac{1}{2} E_p = -\frac{1}{2} \frac{GM \cdot m}{r} = -\frac{1}{2} \cdot \frac{g_0 R_T^{\cancel{2}} \cdot m}{3 \cancel{R_T}} = -\frac{1}{6} g_0 \cdot R_T \cdot m$$

$$\boxed{E_m = -\frac{1}{6} \cdot 9{,}8 \cdot 6{,}4 \times 10^6 \times 100 \simeq -1{,}05 \times 10^9\ J}$$ → Sistema lligat a l'atracció gravitatòria terrestre

8) d) $E_{total} = W_{ext} = \Delta E_m = E_{m\,enllaç} - E_m(R_T) = E_m(r) - E_p(R_T)$

$$W_{ext} = \Delta E_m = -\frac{1}{2}\frac{GMm}{r} - \left(-\frac{GM\cdot m}{R_T}\right) = -\frac{1}{2}\frac{GMm}{r} + \frac{GM\cdot m}{R_T}$$

$$W_{ext} = GMm\left(\frac{1}{R_T} - \frac{1}{2r}\right) = GMm\left(\frac{1}{R_T} - \frac{1}{2.3R_T}\right) =$$

$$W_{ext} = \frac{GMm}{R_T}\left(\frac{1}{1} - \frac{1}{6}\right) = \frac{g_0\cdot R_T^{\cancel{2}}\cdot m}{\cancel{R_T}}\cdot\frac{5}{6}$$

$$\boxed{W_{ext} = 9{,}8\cdot 6{,}4\times10^6\times 100\cdot\frac{5}{6} \simeq 5{,}23\times10^9\ J}$$

e) Just després del llançament (R_T) $\xleftrightarrow[\text{camp gravitatori conservatiu}]{E_m = ct}$ A l'alçada desitjada ($r \Rightarrow h = 2R_T$) sense orbitar

$$E_{m\,0} = \boxed{E_p(R_T) + E_{c0\,\text{llançament}}} = E_{m(h)} = \boxed{\underset{(v_f = 0)}{E_p(r)}}$$

$$\boxed{E_{c0} = E_p(r) - E_p(R_T)} \; ; \; \boxed{-\Delta E_c = +\Delta E_p}$$

Ha transformat tota l'Ec en Ep en la òrbita nova.

$$E_{c0} = -\frac{GMm}{r} \overset{(--)}{+} \frac{GMm}{R_T}$$

$$E_{c0} = GMm\left(\frac{1}{R_T} - \frac{1}{r}\right) = g_0\cdot R_T^{\cancel{2}}\cdot m\left(\frac{1}{\cancel{R_T}} - \frac{1}{3\cancel{R_T}}\right) = g_0 R_T\cdot m\cdot\frac{2}{3}$$

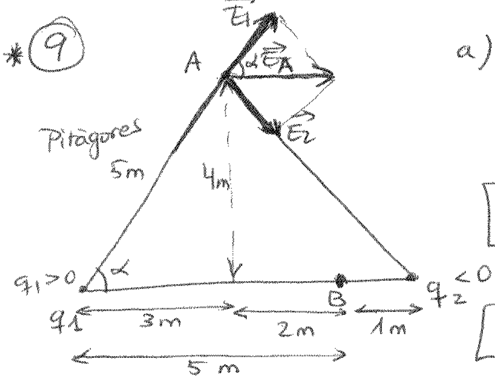
$$\boxed{E_{c0} = 9{,}8\cdot 6{,}4\cdot10^6\cdot 10^2\cdot\frac{2}{3} = 4{,}18\times10^9\ J}$$

$$E_{c0} = \frac{1}{2}m v_0^2 \; ; \; \boxed{v_0 = \sqrt{\frac{2E_{c0}}{m}}} = 9144{,}8\ m/s$$

velocitat de llançament

$$\boxed{\simeq 9{,}1\times10^3\ m/s}$$

*(9)



a) $\boxed{V_A = V_{A1} + V_{A2}} =$

$$V_A = \frac{K q_1}{r_{A1}} + \frac{K q_2}{V_{A2}} = K\left(\frac{q_1}{V_{A1}} + \frac{q_2}{V_{A2}}\right)$$

$$\boxed{V_A = 9 \cdot 10^9 \left(\frac{+20 \cdot 10^{-9}}{5} - \frac{20 \cdot 10^{-9}}{5}\right) = 0\,V}$$

$$\boxed{V_B = V_{B1} + V_{B2}} = \frac{K q_1}{\underset{= 5m}{r_{B1}}} + \frac{K q_2}{\underset{= 1m}{r_{B2}}}$$

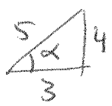
$$V_B = 9 \cdot 10^9 \left(\frac{+20 \cdot 10^{-9}}{5} - \frac{20 \cdot 10^{-9}}{1}\right) =$$

$\boxed{V_B = 9 \cdot 20 \cdot \left(\frac{1}{5} - 1\right) = -144\,V}$ Em caldrà després.

b) $\vec{E_A} = \vec{E_1} + \vec{E_2}$; $|\vec{E_1}| = |\vec{E_2}| = \left|\frac{K q_1}{r_{A1}^2}\right| = \frac{9 \cdot 10^9 \cdot 20 \cdot 10^{-9}}{5^2}$

$$|\vec{E_1}| = |\vec{E_2}| = \frac{180}{25} = 7{,}2\,N$$

. Les components verticals s'anul·len per simetria:
$E_{Ay} = |\vec{E_{1y}}| - |\vec{E_{2y}}| = 0\,N/C$. Llavors, només cal calcular les components horitzontals:
$E_{1x} = E_{2x} = |\vec{E_1}| \cdot \cos\alpha = 7{,}2 \cdot \frac{3}{5} = 4{,}3$



$\cos\alpha = \frac{3}{5}$, així del vector resultant:

$$E_{Ax} = 2 E_{1x} = 2 \cdot 4{,}3 = 8{,}6\,N/C \text{ (mòdul)}$$

$$\boxed{\vec{E_A} = 8{,}6\,\hat{\imath}\ \left(N/C\right)} = E_{Ax}\hat{\imath} + \underset{0}{\cancel{E_{Ay}}}\hat{\jmath}$$

. Vector horitzontal cap a la dreta.

c) $\vec{F_E} = q' \cdot \vec{E} = 5 \cdot 10^{-6} \cdot 8{,}6\,\hat{\imath} = 4{,}3 \cdot 10^{-5}\,\hat{\imath}\ (N)$

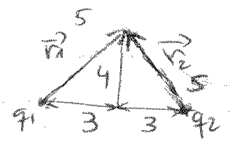
$\overrightarrow{F_E}$ (també cap a la dreta i horitzontal ja que la càrrega és positiva)

d) $\underset{B \to A}{W_{ext}} = +\Delta E_p = q' \cdot \Delta V = q'(V_A - \underbrace{V_B}) = 4 \cdot 10^{-9}(0-(-144)) \Rightarrow$ calculat a l'apartat (a)

$$\boxed{\underset{B \to A}{W_{ext}} = +5{,}76 \times 10^{-7}\ J}$$

Es un procés NO espontani: cal fer un treball exterior en "contra" de les forces del camp (forces electrostàtiques) per tal d'aconseguir traslladar $q' = 4 \cdot 10^{-9}C > 0$ (positiva) d'un punt de potencial negatiu a un punt de potencial nul $V_A = 0$. Les càrregues positives es mouen espontàniament capa potencials decreixents i aquí fem el procés contrari. Concorda amb el fet de que allunyem una $q' > 0$ d'una $q_2 < 0$ les quals s'atreien...

b)* Per vectors unitaris:



$\vec{r_1} = 3\hat{\imath} + 4\hat{\jmath}$; $\hat{u}_1 = \frac{\vec{r_1}}{|\vec{r_1}|} = \frac{3\hat{\imath}+4\hat{\jmath}}{\sqrt{3^2+4^2}} = \frac{3\hat{\imath}+4\hat{\jmath}}{5}$

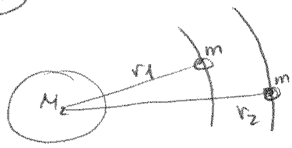
$\vec{r_2} = -3\hat{\imath} + 4\hat{\jmath}$; $\hat{u}_2 = \frac{-3\hat{\imath}+4\hat{\jmath}}{5}$

$\vec{E_1} = \frac{Kq_1}{r_1^2}\hat{u}_1 = \frac{9 \cdot 10^9 \cdot 20 \cdot 10^{-9}}{5^2}\left(\frac{3\hat{\imath}+4\hat{\jmath}}{5}\right) = 4{,}3\hat{\imath} + 5{,}8\hat{\jmath}\ (N/C)$

$\vec{E_2} = \frac{Kq_2}{r_2^2}\hat{u}_2 = \frac{9 \cdot 10^9 \cdot (-20 \cdot 10^{-9})}{5^2}\left(\frac{-3\hat{\imath}+4\hat{\jmath}}{5}\right) = 4{,}3\hat{\imath} - 5{,}8\hat{\jmath}\ (N/C$

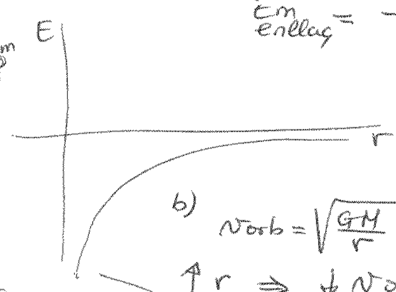
$\vec{E_1} + \vec{E_2} = \boxed{\vec{E_A} = 8{,}6\hat{\imath}\ (N/C)}$ horitzontal cap a la dreta!

*(10)



$m_1 = m_2$ (idèntics) a) VERITABLE

$\underset{enllaç}{E_m} = -\frac{1}{2}\frac{GMm}{r} = +\frac{1}{2}E_p$.



$\uparrow r \ \uparrow E_m$ (són números negatius)

b) $v_{orb} = \sqrt{\frac{GM}{r}}$ $(F_c = F_G)$

$\uparrow r \Rightarrow \downarrow v_{orbital}$ FALSA

c) $E_p = \frac{Kq_+ \cdot q_-}{r} < 0$ $(q_+ \cdot q_- < 0)$ $\uparrow r \Rightarrow \uparrow E_p$ VERITABLE (són números negatius)

d) VERITABLE (Teoria). Significaria dos valors de $\vec{E}$ en un mateix punt.

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