On Clifford algebras and related to them finite groups and group algebras

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Abstract: Albuquerque and Majid [7] have shown how to view Clifford algebras $C\ell_{p,q}$ as twisted group rings $\mathbb{R}^t[(\mathbb{Z}_2)^n]$ whereas Chernov [10] has observed that for each Clifford algebra $C\ell_{p,q}$ there exists a finite 2-group G of order 2^{p+q+1} such that $C\ell_{p,q}$ is a homomorphic image of its group algebra $\mathbb{R}[G]$. Abłamowicz and Fauser [4–6] have introduced a special *transposition automorphism* $T_\varepsilon^{\,\tilde{}}$ of $C\ell_{p,q}$ and have studied various subgroups of Salingaros vee groups $G_{p,q} \subset C\ell_{p,q}$ in relation to spinor representations of $C\ell_{p,q}$. Depending on the isomorphism class of $C\ell_{p,q}$, every Salingaros vee group belongs to one of five families of central products of extra-special dihedral group D_8, the quaternionic group Q_8 and $\mathbb{Z}_2 \times \mathbb{Z}_2$, or $\mathbb{Z}_4$ (Brown [9], Salingaros [26–28], Varlamov [30]). The purpose of this talk is to bring these concepts together in an attempt to relate algebraic properties of Clifford algebras to the properties of these groups and their group rings.

Keywords: 2-group, central product, Clifford algebra, extra-special group, group algebra, transposition, Salingaros vee group

References

[1] R. Abłamowicz: *Computation of non-commutative Gröbner bases in Grassmann and Clifford algebras*. Adv. Applied Clifford Algebras 20 (3–4) (2010), 447–476.

[2] R. Abłamowicz, R. and B. Fauser *Mathematics of `CLIFFORD`: A Maple package for Clifford and Grassmann algebras*, Adv. Applied Clifford Algebras 15 (2) (2005), 157–181.

[3] R. Abłamowicz and B. Fauser: `GfG` - *Groebner for Grassmann - A Maple 12 Package for Groebner Bases in Grassmann Algebras*, http://math.tntech.edu/rafal/GfG12/ (2010).

[4] Abłamowicz, R. and B. Fauser: *On the transposition anti-involution in real Clifford algebras I: The transposition map*, Linear and Multilinear Algebra 59 (12) (2011), 1331–1358.

[5] R. Abłamowicz and B. Fauser: *On the transposition anti-involution in real Clifford algebras II: Stabilizer groups of primitive idempotents*, Linear and Multilinear Algebra 59 (12) (2011), 1359–1381.

[6] R. Abłamowicz and B. Fauser: *On the transposition anti-involution in real Clifford algebras III: The automorphism group of the transposition scalar product on spinor spaces*, Linear and Multilinear Algebra 60 (6) (2012), 621-644.

[7] H. Albuquerque and S. Majid: *Clifford algebras obtained by twisting of group algebras*, J. Pure Applied Algebra 171 (2002), 133–148.

[8] R. Bonezzi, N. Boulanger, E. Sezgin, P. Sundell: *Frobenius-Chern-Simons gauge theory*, `arXiv: 1607.00726v1`, July 4, 2016.

[9] Z. Brown: *Group Extensions, Semidirect Products, and Central Products Applied to Salingaros Vee Groups Seen As 2-Groups*, Master Thesis, Department of Mathematics, TTU, Cookeville, TN, December 2015.

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[10] V. M. Chernov, *Clifford Algebras as Projections of Group Algebras*, in *Geometric Algebra with Applications in Science and Engineering*, E. B. Corrochano and G. Sobczyk, eds., Birkhäuser, Boston (2001), 461–476.

[11] C. Chevalley: *The Algebraic Theory of Spinors*, Columbia University Press, New York, 1954.

[12] L. L. Dornhoff, *Group Representation Theory: Ordinary Representation Theory*, Marcel Dekker, Inc., New York, 1971.

[13] The GAP Group, *GAP – Groups, Algorithms, and Programming, Version 4.7.8*; 2015, http://www.gap-system.org.

[14] D. Gorenstein, *Finite Groups*, 2nd. ed., Chelsea Publishing Company, New York, 1980.

[15] J. Helmstetter: *Groupes de Clifford pour de formes quadratiques de rang quelconque.* C. R. Acad. Sci. Paris (1977), 175–177.

[16] J. Helmstetter: *Algèbres de Clifford et algèbres de Weyl*, Cahiers Math. 25, Montpellier, 1982.

[17] G. James and M. Liebeck, *Representations and Characters of Groups.* Cambridge University Press, 2nd ed., 2010.

[18] T.Y. Lam, *The Algebraic Theory of Quadratic Forms*, Benjamin, London, 1980.

[19] P. Lounesto: *Clifford Algebras and Spinors.* 2nd ed. Cambridge University Press, Cambridge, 2001.

[20] K. D. G. Maduranga, *Representations and Characters of Salingaros' Vee Groups*, Master Thesis, Department of Mathematics, TTU, May 2013.

[21] K. D. G. Maduranga and R. Abłamowicz: *Representations and characters of Salingaros' vee groups of low order*, Bull. Soc. Sci. Lettres Łódź Sér. Rech. Déform. 66 (1) (2016), 43–75.

[22] C. R. Leedham-Green and S. McKay, *The Structure of Groups of Prime Power Order*, Oxford University Press, Oxford, 2002.

[23] S. Majid, *Foundations of Quantum Group Theory*, Cambridge University Press, Cambridge, 1995.

[24] D. S. Passman, *The Algebraic Structure of Group Rings*, Robert E. Krieger Publishing Company, 1985.

[25] J. J. Rotman, *Advanced Modern Algebra*, 2nd. ed., American Mathematical Society, Providence, 2002.

[26] N. Salingaros, *Realization, extension, and classification of certain physically important groups and algebras*, J. Math. Phys. 22 (1981), 226–232.

[27] N. Salingaros, *On the classification of Clifford algebras and their relation to spinors in n dimensions*, J. Math. Phys. 23 (1) (1982), 1–7.

[28] N. Salingaros, *The relationship between finite groups and Clifford algebras*, J. Math. Phys. 25 (4) (1984), 738–742.

[29] V. V. Varlamov: *Universal coverings of the orthogonal groups.* Adv. in Applied Clifford Algebras 14 (1) (2004), 81–168.

[30] V. V. Varlamov, *CPT Groups of Spinor Fields in de Sitter and Anti-de Sitter Spaces*, Adv. Appl. Clifford Algebras 25 (2) (2015) 487–516.