Grassmann’s powerful but largely undefined approach to affine and projective geometries has roots in Möbius’ barycentric calculus [1]. In his appraisal of the former’s work, Peano showed explicitly the relation of barycentric coordinates with Cartesian coordinates [2]. In our Treatise ([3] p. 33) we went beyond Peano’s work by displaying the advantage of barycentric coordinates in dealing with the main theorems of projective geometry (Desargues, Pappus, etc.). By giving the equations of lines and planes with barycentric coordinates, we explain the geometric duality in a purely algebraic way ([3] p. 43, [4]). The generalization of the barycentric coordinates leads to projective frames and coordinates, which allows us to work with the whole projective geometry of an $n$-dimensional space without defining the projective space $\mathbb{P}^n$ as projection of an $n+1$ dimensional space. We will also display the advantages of expressing the equations of quadrics with projective coordinates of the three-dimensional space. According to Grassmann ([5], [6] p. 385), the product of two points is a line, the product of three points is their plane and the product of four points is the whole space. In the same way, the successive products of dual points in the dual space generate geometric elements having decreasing dimensions [7]. Then, Grassmann’s products of points and dual points will be identified respectively with the operators $\text{join}$ and $\text{meet}$ of the projective geometry. Finally, let us emphasize that all these conclusions can be generalized to $n$-dimensional spaces.

REFERENCES