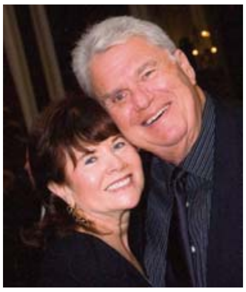
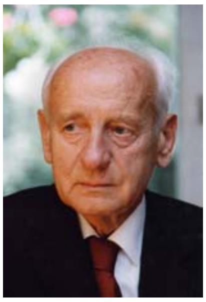
Alterman Conference on Geometric Algebra

August 4th to 6th, 2016   Braşov (Romania)



BOOKLET OF ABSTRACTS



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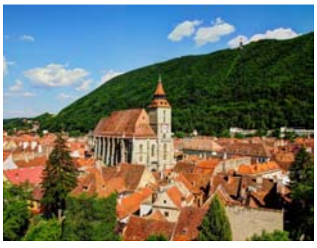

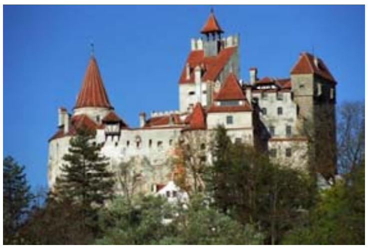


This booklet was printed at the Transilvania University of Brasov in 2016.

Dear Colleagues,

Together with the other organizers, I want to thank Mr. Eric Alterman for making possible this Conference and Summer School.

We welcome all participants, and specially those among you who made the extra effort to prepare a presentation, and to follow custom-made advice from the Scientific Committee.

We celebrate the quality and novelty of the abstracts submitted. Some of the contributions attracted great interest from members of the Scientific Committee; our warmly felt thanks to you. We are also glad because some new talks will be presented at the conference, and we are sorry for some authors who wished to come and cannot attend the Conference.

Finally, there will not be parallel sessions. Everybody could thus hear talks by everybody else. Allow us then to make a recommendation, or a request if you will. It may help if, without sacrificing contents of your papers, you introduce highlights that are not meant for experts. We shall thus achieve that everybody will take something home from everybody else's talks. Please remind me (Jose) to follow my own advice, at least during my lectures.

Abstracts appear in alphabetic order. Occasionally, it may not look that way because there are cultures where names and given names do not go in the same order as in the West.

I will be pleased to meet all of you in person in Brasov.

Cordially,

Jose G. Vargas, Chairman. On behalf of the Organizing Committee.

July 2016

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Special topic: *Grassmann's legacy with emphasis on the projective geometry*

General section:

Special topic:

Grassmann's legacy with emphasis on the projective geometry

talk 1

PROJECTIVE, CLIFFORD AND GRASSMANN ALGEBRAS AS COMPLEMENTARY GRADED ALGEBRAS

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In this talk we will establish projective algebra Λ_n together with the complementary graded Clifford algebra Γ_n and compare both (a) to what is usually known as Grassmann algebra and (b) to Grassmann algebra in the approach of John Brown. [1]

The 2^n-dimensional projective algebra $\Lambda_n(+,\cdot,\wedge,\vee)$ and the 2^n-dimensional complementary graded Clifford algebra $\Gamma_n(+,\cdot,\ ,*)$ both carry the imprint of a graded algebra twice, i. e. they have a dual axiomatic structure. Projective algebra is the more fundamental concept than the complementary graded Clifford algebra, since any complementary graded Clifford algebra shows also the structure of projective algebra whereas projective algebra is standing on its own.

John Browne used the term *Grassmann algebra* in [1] to describe the body of algebraic theory and results based on Graßmann's *Ausdehnungslehre* from 1844 and 1862. This Grassmann algebra shows a dual axiomatic structure as projective algebra and complementary graded Clifford algebra do. We will compare Grassmann algebra in the approach of John Browne with the complementary graded algebras Λ_n and Γ_n.

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talk 2

PROJECTIVE GEOMETRY WITH PROJECTIVE ALGEBRA

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Many analytic descriptions for projective geometry are not representing the *complete* wealth of projective geometry such as it is know from *synthetic* projective geometry. In most of the analytic descriptions the basic elements are reduced to points or to points and hyperplanes, but do not, for example, include the lines and linear complexes of space (in the form of basic elements). A further failure often is that the principle of duality is not reflected by the analytic description.

In order to overcome these boundaries, the 2^n-dimensional projective algebra $\Lambda_n(+,\cdot,\wedge,\vee)$ was developed.

This talk will provide a system of axioms for projective geometry $\mathscr{P}_n$ in terms of projective algebra Λ_n. Concepts of projective geometry such as the principle of duality, primitive geometric forms, the cross ratio of four basic elements and projective transformations will be determined in terms of projective algebra.

The above mentioned system of axioms for projective geometry will be compared to other approaches to projective geometry. [1, 2, 3, 4, 5]

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PEANO READER OF H. GRAßMANN'S *AUSDEHNUNGSLEHRE*

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In this talk I propose an analysis of Peano's studies about the geometric calculus. In the volume of 1888 (*Geometric Calculus according to H. Grassmann's Ausdehnungslehre preceded by the operations of deductive logic*), Peano presents Grassmann's ideas (*Ausdehnungslehre*, 1844) in an original way: he gives an Euclidean interpretation to the fundamental Grassmannian notions. Hence by means of his geometric calculus, Peano is able to show theorems of projective geometry. Therefore, Peano's geometrical calculus (which has an intrinsic mathematical interest in order to the applications to the geometry and to the mechanics) has an implicit foundational role. The disciple of Peano who devoted himself above all to the studies of geometric calculus was Cesare Burali Forti (1861-1931); but also Filiberto Castellano (1860-1919), Tommaso Boggio (1877 - 1963) and Mario Pieri (1860-1904) took an interest in the subject.

talk

THE AFFINE AND PROJECTIVE GEOMETRIES FROM GRASSMANN'S POINT OF VIEW

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Grassmann's powerful but largely undefined approach to affine and projective geometries has roots in Möbius' barycentric calculus [1]. In his appraisal of the former's work, Peano showed explicitly the relation of barycentric coordinates with Cartesian coordinates [2]. In our Treatise ([3] p. 33) we went beyond Peano's work by displaying the advantage of barycentric coordinates in dealing with the main theorems of projective geometry (Desargues, Pappus, etc.). By giving the equations of lines and planes with barycentric coordinates, we explain the geometric duality in a purely algebraic way ([3] p. 43, [4]). The generalization of the barycentric coordinates leads to projective frames and coordinates, which allows us to work with the whole projective geometry of an n-dimensional space without defining the projective space $\mathbb{PR}_n$ as projection of an $n+1$ dimensional space. We will also display the advantages of expressing the equations of quadrics with projective coordinates of the three-dimensional space. According to Grassmann ([5], [6] p. 385), the product of two points is a line, the product of three points is their plane and the product of four points is the whole space. In the same way, the successive products of dual points in the dual space generate geometric elements having decreasing dimensions [7]. Then, Grassmann's products of points and dual points will be identified respectively with the operators *join* and *meet* of the projective geometry. Finally, let us emphasize that all these conclusions can be generalized to n-dimensional spaces.

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talk

OF GRASSMANNIAN ALGEBRAS AND THE ERLANGEN PROGRAM, WITH EMPHASIS ON PROJECTIVE GEOMETRY

José G. Vargas [a]

[a] PST Associates

Grassmann's legacy is certainly constituted by his many revolutionary concepts and by exterior algebra, rightly attributed to him and which sometimes bears his name. But he also put a foot in the door of Cliifford algebra and, to quote É. Cartan, he also created *a very fruitful geometric calculus* —specially for projective geometry— where both points and vectors pertain to the *first or primitive class* [1]. Grassmann did his work [2] during the golden age of synthetic geometry, which also was the stone age of the algebraic foundations of projective geometry. As we shall show, these foundations are subordinate to those of affine geometry, which is the reason why É. Cartan developed his general theory of connections starting not with the Euclidean or projective ones, but with affine connections. The same will be the case here for the corresponding elementary or Klein geometries, which the theory of the different connections generalizes [3].

The use of algebra that respects the equivalence of all points in affine geometry —thus the absence of a "zero point"— leads to the concept of canonical affine frame bundle, where the frames are constituted by a point and a vector basis. But bundles of frames made of points or of lines or, in dimension n, of linear varieties of dimension $(n-1)$, may also be used in affine geometry [4]. This leads us to consider the relation of frame bundles to Klein geometries.

The representation up to a proportionality constant of projective transformations as homographies, which constitute the projective group of matrices, almost fits the Erlangen program. But the subgroup that leaves a point unchanged —essential in Klein geometries— and the matrix representation of the affine group are typically overlooked. So has been, therefore, the issue of what synthetic projective transformations are directly related to the post-affine entries in the homographies. We exhibit the subgroup of such transformations and show that the proper homologies —i.e. not involving elements at infinity— are directly related to those entries.

We re-interpret from the canonical frame bundle González's version of Möbius-Grassmann-Peano theory, the usefulness of that bundle being enriched in the process. Thus, his special barycentric coordinates now also belong to a theory of moving frames where one includes "frames that do not move". Improper elements, arising from the use of homogeneous coordinates, are not needed if duality is not taken too far, as when one replaces the statement that "parallel lines do not intersect" with the statement that "they intersect at a point at infinity". It is worth noting that the line at infinity is dual to the centroid of a triangle, which is not a special point. So, duality is a very important correspondence, but does not respect the equivalence of all points (unless, of course, we were to create an unnecessary superstructure that mimicked the bundles of frames). Thus González's treatment of Grassmann's system for projective geometry takes it closer to the theory of the moving frame. Of course, there is nothing moving in this case, since nothing needs to do so in the Klein geometries; only their Cartanian generalizations need that the frames "move".

We proceed to briefly summarize Cartan's derivation of the equations of structure of projective connections [5].

Finally, the Kähler calculus[6] can claim to have Grassmann in its ascendancy. We shall illustrate how it blends Clifford algebra and exterior calculus.

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General section

talk

ON CLIFFORD ALGEBRAS AND RELATED TO THEM FINITE GROUPS AND GROUP ALGEBRAS

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Abstract: Albuquerque and Majid [7] have shown how to view Clifford algebras $Cl_{p,q}$ as twisted group rings $\mathbb{R}^t[(\mathbb{Z}_2)^n]$ whereas Chernov [10] has observed that for each Clifford algebra $Cl_{p,q}$ there exists a finite 2-group G of order 2^{p+q+1} such that $Cl_{p,q}$ is a homomorphic image of its group algebra $\mathbb{R}[G]$. Abłamowicz and Fauser [4, 5, 6] have introduced a special *transposition automorphism* T_ε^- of $Cl_{p,q}$ and have studied various subgroups of Salingaros vee groups $G_{p,q} \subset Cl_{p,q}$ in relation to spinor representations of $Cl_{p,q}$. Depending on the isomorphism class of $Cl_{p,q}$, every Salingaros vee group belongs to one of five families of central products of extra-special dihedral group D_8, the quaternionic group Q_8 and $\mathbb{Z}_2 \times \mathbb{Z}_2$, or $\mathbb{Z}_4$ (Brown [9], Salingaros [26, 27, 28], Varlamov [30]). The purpose of this talk is to bring these concepts together in an attempt to relate algebraic properties of Clifford algebras to the properties of these groups and their group rings.

Keywords: 2-group, central product, Clifford algebra, extra-special group, group algebra, transposition, Salingaros vee group

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talk

POLYNOMIAL PERMUTATIONS OF FINITE RINGS AND FORMATION OF LATIN SQUARES

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Combinatorial designs have wide applications in various fields, including coding theory and cryptography. Many examples of combinatorial designs can be listed like linked design, balanced design, one-factorization etc. Latin square is one such combinatorial concept. In this talk, we have considered different types of permutation polynomials over some finite rings. Over finite rings, we have observed that univariate permutation polynomials permute the ring elements whereas bivariate permutation polynomials form Latin squares. The Latin squares formed thus by permutation polynomials over finite rings are discussed with respect to various Latin square properties.

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SCALED PLANAR NEARRINGS

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Abstract: A nearring $(N,+,\cdot)$ is a structure similar to a ring, but without the request to be additively commutative and with just one of the distributive laws. In this work, we deal with a special nearring structure called planar nearring and introduce a new structure called scaled planar nearring. We prove that every scaled planar nearring is zero symmetric and deduce some structure theorems. We illustrate that the scaling factor of the scaled planar nearring can be used to understand ideas from projective geometry. Let $(F,+,\cdot)$ be a finite nearfield and $N = F\times F$. It is well known that if F is a field then the affine plane $(N,\boldsymbol{L},\varepsilon)$ is desarguesian and that finite nearfields (which are not fields) can be used to construct non-desarguesian planes. We demonstrate that a suitable choice of scaling factor can be made to construct desarguesian planes or non-desarguesian planes.

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talk

HIGHER DIMENSIONAL REPRESENTATIONS OF SL_2 AND ITS REAL FORMS VIA PLÜCKER EMBEDDING

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In the present paper we study the inclusion of the complex Lie algebra $\mathfrak{sl}_2 \cong \mathfrak{so}_3 \subset \mathfrak{so}_n$ realized as a Plücker embedding, and thus, attempt to construct higher dimensional representations of the real forms of SO_3 in terms of $\mathsf{SO}(n)$ and $\mathsf{SO}(p,q)$ transformations, beyond the standard block-matrix realization. Moreover, we consider Euler and Wigner type decompositions in this setting and show how the Plücker relations appear in a natural way. Explicit examples are provided for $n = 3$, 4 and 5 in the context of special relativity, classical and quantum mechanics.

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A SYSTEMATIC CONSTRUCTION OF REPRESENTATIONS OF QUATERNIONIC TYPE

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The appearance of quaternions in representation theory is usually by accident, both poorly understood yet thought to be deeply meaningful [5, 6]. Examples are the representations of root systems in 3D and 4D in terms of (pure) quaternions, as well as representations of quaternionic type of the polyhedral and other groups.

I have demystified the former in previous work, showing that 4D root systems are induced from 3D root systems in complete generality; in particular, the 3D root systems can only be realised in terms of pure quaternions when the corresponding reflection group contains the inversion [1, 2, 3]. The emergence of the 4D root systems hinges on the Clifford algebra of 3D, or rather its even subalgebra. The spinors describing rotations in 3D (from even products of the reflection generating root vectors) can be endowed with a well-known 4D Euclidean distance. The axioms of a root system are then easily satisfied: firstly, via the Euclidean metric a 3D spinor group can be treated as a collection of vectors in 4D; secondly, Clifford spinorial methods provide a double cover of rotations such that the 4D collection of vectors contains the negatives of those vectors, and thirdly with respect to the 4D Euclidean distance and using some other properties of spinors, the collection of 4D vectors is closed under reflections amongst themselves. These representations of root systems in terms of quaternions therefore systematically hinge purely on the geometry of 3D and the accident that the even subalgebra, i.e. the spinors, is quaternionic.

Here I discuss systematically the representation theory of the intimately related polyhedral groups [4, 2]. The 8D Clifford algebra of 3D space allows one to easily define various representations: the trivial one, parity, the usual 3×3 rotation matrix representation acting on a 3D vector achieved by sandwiching a vector with the corresponding versor, or the 8×8 representation of the group elements as reshuffling the multivector components in the whole 8D algebra under multivector multiplication. The representations we will focus on, however, are those defined by acting with any spinor on another general spinor. This reshuffles the components of the general spinor, which can also be expressed as a 4×4 matrix acting on the spinor in column format. It is not surprising that again because of the quaternionic nature of the even subalgebra the representations of quaternionic type of the polyhedral groups arise naturally and geometrically in a systematic way. Both observations therefore demystify quaternionic phenomena as consequences of 3D geometry, and in particular the ‘mysterious deep significance’ is simply provided by their spinorial nature – both simple yet underappreciated.

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A CONFORMAL GEOMETRIC ALGEBRA CONSTRUCTION OF THE MODULAR GROUP

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I will discuss a new construction of the modular group [1]. My interest in the modular group stems from recent Moonshine observations, relating string theory, finite simple groups and (mock) modular forms [2, 3]. The modular group is a subgroup of the 2D conformal group and I use the conformal model in Geometric Algebra with the corresponding Clifford realisations of the conformal group [4, 5] to construct a new realisation of the modular group. The double cover of the modular group is the braid group; of course this Clifford construction in fact provides a double cover of the conformal and thus modular groups, and we will discuss the relation with the braid group.

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talk

THE SIMPLEST NON – ASSOCIATIVE GENERALIZATION OF SUPERSYMMETRY

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Non – associative generalization of supersymmetry is offered [1]. 3– and 4–points associators for supersymmetric generators are considered. On the basis of zero Jacobiators for three supersymmetric generators we have obtained the simplest form of 3–point associators. The connection between 3– and 4–point associators are considered. On the basis of this connection 4–point associators is obtained. The Jacobiators for the product of four supersymmetric generators are calculated. We discuss possible physical sense of numerical coefficients presented on the RHS of associators. The possible connection between supersymmetry, hidden variables and non – associativity is discussed.

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talk

GA AND GC APPLIED TO PRE-SPATIAL ARITHMETIC SCHEME TO ENHANCE MODELING EFFECTIVENESS IN BIOPHYSICAL APPLICATIONS

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The classic vector analysis is unable to describe the Relativity and Quantum Field Theory so that an increasing attention to the geometric algebra (GA) and geometric calculus (GC) has been paid. We present an exponential, pre-spatial arithmetic scheme ("all-powerful scheme") to overcome the limitation of the traditional probabilistic modeling veil opacity in complex arbitrary multiscale system modeling. Most recent approaches take into consideration multivariate cumulative distribution function and all current implementations rely on statistic and probabilistic analysis only. To grasp more reliable representation of experimental reality researchers and scientists need two intelligently articulated hands: both statistical and combinatorical approaches synergistically articulated by natural coupling. We need to consider a model not only on the statistical manifold of model states but also on the combinatorical manifold of low-level discrete, elementary phased generators. *CICT* (computational information conservation theory) [1] new awareness of a discrete HG (hyperbolic geometry) subspace (reciprocal-space, RS) of coded heterogeneous hyperbolic structures, underlying the familiar $\mathbb{Q}$ Euclidean direct-space (DS) surface representation, shows that any natural number n in $\mathbb{N}$ has associated a specific, non-arbitrary extrinsic or external phase relationship that we have to take into account to full conserve overall system component information content by computation in DS [2]. Traditional $\mathbb{Q}$ numeric system elementary arithmetic long division remainder sequences can be interpreted as combinatorically Optimized Exponential Cyclic Sequences encoding hyperbolic geometric structured information, as points on a discrete Riemannian manifold, under HG metric [3]. They can encode both modulus and extrinsic phase information, which elementary phased generator intrinsic phase can be computed from. Phased generators can even offer a solution to parallel transport problems, taking into account associated components extrinsic phase relationships and their consonant or dissonant behavior. We show how to unfold the full information content of Rational numeric representation (nano-microscale discrete representation) and to relate it to a continuum framework (meso-macroscale) effectively. GA and GC unified mathematical language with *CICT* can offer a competitive and effective "Science 2.0" [2] universal arbitrary multiscale computational framework for biophysical applications.

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talk

ON MATRIX REPRESENTATIONS OF GEOMETRIC (CLIFFORD) ALGEBRAS

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The representations of geometric (Clifford) algebras with square real matrices [1, 2] are reviewed in order to see whether some advantage can be gained when considering them from an arithmetic point of view [3], meaning without resorting to algebraic structure. Many isometries such as rotations [4, 5, 6] and Lorentz transformations [7] are written as similarity transformations of matrices, which will be chosen as the general definition of isometry. Then, it will be deduced in which geometric algebras all the known isometries as well as the transformations that could be considered isometries can be written as similarity transformations. Since similar matrices have the same characteristic polynomial [8], the norm of every element of a geometric algebra should be defined from some combination of its coefficients. It is proposed to define the norm of every element of a geometric algebra as the n^{th}-root of the absolute value of the determinant of its matrix representation with dimension $n \times n$, which is the independent term of the characteristic polynomial. Some examples of the usefulness of this definition will be provided in order to confirm its consistency and generality.

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talk

INVERSE KINEMATICS BASED ON BINOCULAR VISION

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Our goal is to grasp an object lying in 3D space using inverse kinematics. Using image analysis with two cameras located in independent axes we calculate the position of the reference line and grasp the object lying in this line. We use some advantages of CGA in our particular setting.

Classically, for modeling a 3D robot, the whole CGA (i.e. $\mathscr{C}l(4,1)$) is used, where the embedding $\mathbb{R}^3 \to \mathbb{K}^4 \to \mathbb{R}^{4,1}$ is considered, where $\mathbb{K}^4$ is a null cone and $\mathbb{R}^{4,1}$ is a Minkowski space. Consequently, the embedding is of the form $c(x) = x + \frac{1}{2}x^2 e_\infty + e_0$. Note that e_0 and e_∞ play the role of the origin and the infinity, respectively.

CGA provides a set of basic geometric entities to compute with, namely points, spheres, planes, circles, lines, and point pairs. These entities have two algebraic representations, IPNS and OPNS which are duals of each other by convolution $(\cdot)^*$. In OPNS representation we can handled with spheres $c(x_1) \wedge c(x_2) \wedge c(x_3) \wedge c(x_4)$, planes $c(x_1) \wedge c(x_2) \wedge c(x_3) \wedge e_\infty$, circles $c(x_1) \wedge c(x_2) \wedge c(x_3)$ and lines $c(x_1) \wedge c(x_2) \wedge e_\infty$. In OPNS, the sphere is represented as $S = c(x) - \frac{1}{2}r^2 e_\infty$, where $c(x)$ is a center point and r is a radius. Note that the properties and definitions of conformal geometric algebras can be found in e.g. [2].

The problem, we split into two parts:

1. Binocular vision. Using the data obtained from both cameras, the line symmetry is transformed from the image coordinates to camera coordinates, i.e. from $\mathbb{R}^2 \to \mathbb{R}^3$. With the lines L_1 and L_2 in IPNS representation and the camera focuses c_1 and c_2 we create two planes $L_1 \wedge c_1$ and $L_2 \wedge c_2$ in IPNS, such that, the reference line is an intersection of these planes.

1. Inverse kinematics. Our robot has five degrees of freedom obtained by means of the five joint angles $\theta_1, \ldots, \theta_5$. Our goal is to find the joint angles in terms of the target position. In CGA, this inverse kinematics problem can be solved in a intuitive way by the handling of intersections of spheres. For example, let us have two links $P_{i-1}P_i$ and P_iP_{i+1}, i.e. the joint P_i has to lie on the sphere with center point P_{i-1} and on the sphere with center point P_{i+1} with the length of the vectors $\overrightarrow{P_{i-1}P_i}$ and $\overrightarrow{P_iP_{i+1}}$ as the radius respectively. The first sphere is represented by OPNS element $S_{i-1} = P_{i-1} - \frac{1}{2}|\overrightarrow{P_{i-1}P_i}|^2 e_\infty$ and the second by OPNS element $S_i = P_{i+1} - \frac{1}{2}|\overrightarrow{P_iP_{i+1}}|^2 e_\infty$. From the theory it is easy to see that $(S_{i-1} \wedge S_i)^*$ is an OPNS representation of circle, such that P_i belongs to this circle.

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talk

IDEALS IN MATRIX NEARRINGS AND GROUP NEARRINGS

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Let N be a zero symmetric right nearring with identity 1 and N^n denote the direct sum of n copies $(n \geq 2)$ of the underlying group $(N,+)$. We consider the $n \times n$ matrix nearring over N, denoted by $M_n(N)$, generated by the set of functions $\{[a;i,j] : 1 \leq i,j \leq n, a \in N\}$. It is well known that ideals in the base nearring N and related ideals in the corresponding matrix nearring $M_n(N)$ have been extensively studied in $[5,11,13]$. In this talk, some observations have been made on idempotent elements, nilpotent elements in the nearring and the corresponding matrix nearring. In case of a finite group G, with $|G| = n$, the notion of group nearring, defined in [3], is closely related to $M_n(N)$. Few analogue relationships between the ideals of nearring and that of group nearring are presented. For preliminary definitions and results on nearrings, we refer [6, 12].

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talk

TENSOR PRODUCTS OF CLIFFORD ALGEBRAS

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In [2], Jacobson viewed Clifford algebras as tensor products of Clifford algebras of lower dimensions. We develop further this point of view on Clifford algebras. We consider real or complex Clifford algebras as tensor products of Clifford algebras of dimensions 2 and 1 and obtain an analog (in terms of tensor products) of Cartan's classification of real Clifford algebras. In our opinion, the new point of view gives greater flexibility to the theory of Clifford algebras and extends the possibilities of application of the mathematical apparatus of Clifford algebras.

It is proved [1] that the tensor product of any Clifford algebras is isomorphic to a single Clifford algebra over some commutative algebra. It is also proved that any complex or real Clifford algebra Cl(p, q) can be represented as a tensor product of Clifford algebras of the second and first orders. A canonical form of such a representation is proposed.

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talk

COHERENT STATES AND BEREZIN TRANSFORMS ATTACHED TO LANDAU LEVELS

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In general, coherent states $(|x\rangle)_{x\in X}$ are a specific overcomplete family of normalized vectors in the Hilbert space $\mathcal{H}$ of the problem that describes the quantum phenomena and solves the identity of $\mathcal{H}$ as

$$1_{\mathcal{H}} = \int\limits_{X} |x\rangle \langle x| \, d\mu\,(x)\,.$$

These states have long been known for the harmonic oscillator and their properties have frequently been taken as models for defining this notion for other models. We review the definition and properties of coherent states with examples. We construct coherent states attached to Landau levels (discrete energies of a uniform magnetic field) on three known examples of Kähler manifolds X: the Poincaré disk $\mathbb{D}$, the Euclidean plane $\mathbb{C}$ and the Riemann sphere $C\mathbb{P}^1$. After defining their corresponding integral transforms, we obtain characterization theorems for spaces of bound states of the particle. Generalization to $\mathbb{C}^n$ and to the complex unit ball $\mathbb{B}^n$ and $C\mathbb{P}^n$ are also discussed. In these cases, we apply a coherent states quantization method to recover the corresponding Berezin transforms and we give formulae representing these transforms as functions of Laplace-Beltrami operators.

talk

LINEAR 2-NORMED SPACES AND 2-BANACH ALGEBRAS

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The concept of linear 2- normed spaces was introduced by Siegfried Gahler in 1963 [6], which is nothing but a two dimensional analogue of a normed space. This concept had received the attention of a wider audience after the publication of a paper by A. G. White in 1969 entitled 2-Banach spaces [7]. In this talk we would like to present the recent developments in Linear 2-normed spaces and 2-Banach algebras. We introduce the idea of expansive, non-expansive and contraction mappings in linear 2-normed spaces eventually some of its properties are established. The analogous of Banach fixed point theorem for contraction mappings in linear 2- normed spaces is obtained, which leads to the existence of the solution of strong accretive operator equation in linear 2-normed space. Some more analogues results in Linear 2-normed spaces and 2-Banach algebras are obtained.

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FACIAL RECOGNITION USING MODERN ALGEBRA AND MACHINE LEARNING

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Facial Recognition Systems have been garnering attention from various researchers and enthusiasts in recent times, they are being deployed for numerous applications like those in identifying faces from a mass of subjects [1], noise removal from captured images, forensic applications [4], etc. The principle idea is to identify and extract individual features from the image of an individual's face. The RGB image is first converted to grayscale and then in reference to a threshold intensity, are transformed into a binary matrix [6]. The end-point demarcations [7] of individual features, called feature points are identified from the image and the relative distances between relevant points are calculated using a wide range of algebraic functions like Euclid distance, eigenvectors, etc. These distances [5] are stored in the form of vectors and are then transformed as required. The images are classified on the basis of similar distances between concurrent features, and are grouped together under one class. This is represented as a point in a high dimensional space. Recent research is focused on improving the accuracy, efficiency and speed of existing systems. So, in this paper we focus on eigenvalues [3], eigenvectors arising due to factors like covariance matrix, dominant eigenvalues, principal components. The basic purpose of algebra, is to enhance the features, in terms of clarity, and also classification of data, using these features. In this paper, we have achieved, better extraction of features, when compared to artifacts [9], and also classification accuracies have improved when compared to existing literature.

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talk

CLIFFORD ALGEBRA IMPLEMENTATIONS IN MAXIMA

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Maxima is the open source descendant of the first ever computer algebra system and features a rich functionality from a large number of shared packages. While written in Lisp, *Maxima* has its own programming language, based on Lisp. The Maxima language is based on the ideas of functional programming, which is particularly well suited for formal transformations of mathematical expressions. The packages *clifford* and *cliffordan* authored by the presenter, implement Clifford algebras $C\ell_{p,q,r}$ of arbitrary signatures and order. The *clifford* package defines multiple rules for pre- and post-simplification of Clifford products, outer products, scalar products, inverses and powers of Clifford vectors [1]. Using this functionality any combination of products can be put into a canonical representation, for example in the quaternion algebra $C\ell_{0,2}$:

```
mtable1([1, e[1],e[2], e[1] . e[2]]);
```

$$\begin{pmatrix} 1 & e_1 & e_2 & e_1.e_2 \\ e_1 & -1 & e_1.e_2 & -e_2 \\ e_2 & -e_1.e_2 & -1 & e_1 \\ e_1.e_2 & e_2 & -e_1 & -1 \end{pmatrix}$$

$block(declare([a,b,c,d],scalar),cc:a+b*e[1]+c*e[2]+d*e[1].e[2],dd:cinv(cc))$

$$\frac{a-e_1\,b-e_2\,c-(e_1.e_2)\,d}{a^2+b^2+c^2+d^2}$$

The inner product is represented by the operator symbol "|" and the outer (exterior, or wedge) product by the operator symbol "&". For example the sum of the inner and outer products of two elements immediately simplifies into the full Clifford product:

```
a | b + a & b;
```

$$a\cdot b$$

or the Jacobi identity automatically holds for the even-grade multivectors:

```
a & b & c + b & c & a + c & a & b;
```

$$0$$

The presentation will demonstrate applications of *clifford* and *cliffordan* in linear algebra and calculus.

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talk

FOCK REPRESENTATIONS AND DEFORMATION QUANTIZATION OF KÄHLER MANIFOLDS

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The goal of this talk is to construct the Fock representation of noncommutative Kähler manifolds. Noncommutative Kähler manifolds studied here are constructed by deformation quantization with separation of variables. This deformation quantization was given by Karabegov. The algebra of the noncommutative Kähler manifolds contains the Heisenberg-like algebras. Local complex coordinates and partial derivatives of a Kähler potential satisfy the commutation relations between creation and annihilation operators. A Fock space is spanned by a vacuum, which is annihilated by all annihilation operators, and states obtained by acting creation operators on this vacuum. The algebras on noncommutative Kähler manifolds are represented as those of linear operators acting on the Fock space. We call the representation of the algebra Fock algebra. In representations studied here, creation operators and annihilation operators are not Hermitian conjugate with each other, in general. Therefore, the bases of the Fock space are not the Hermitian conjugates of those of the dual vector space. In this case, we call the representation the twisted Fock representation. In this presentation, we construct the twisted Fock representations for arbitrary noncommutative Kähler manifolds given by deformation quantization with separation of variables, and we give a dictionary to translate between the twisted Fock representations and functions on noncommutative Kähler manifolds concretely.

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talk

ON SOME LIE GROUPS CONTAINING SPIN GROUPS IN CLIFFORD ALGEBRA

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We consider 15 different Lie groups in Clifford algebra of arbitrary dimension and signature and prove isomorphisms between these groups and classical matrix Lie groups - symplectic, orthogonal, unitary and linear groups. Also we obtain isomorphisms of corresponding Lie algebras. Information about pseudo-unitary group $Wcl(p,q)$ you can find in [1], [2] and [3]. Several Lie groups are discussed in [4].

Spin group is a subgroup of all considered Lie groups. One of considered Lie groups coincides with group $Spin_+(p,q)$ in the cases of dimensions $n \leq 5$.

We use the notion of the quaternion type [5], [6] in our considerations. New classification based on the notion of the quaternion type helps us to analyse [7] commutators of Clifford algebra elements.

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talk

WHAT THE KAEHLER CALCULUS CAN DO THAT OTHER CALCULI CANNOT

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The progress of mathematics makes it sometimes fashionable to describe physical theory in more modern forms, not necessarily deeper or more understandable. Of particular interest in this regard is relativistic quantum mechanics. Modern versions of it, like through the use of geometric calculus (read Hestenes [1]), may be more appealing than the original version of Dirac theory [2], but the physical contents remains virtually unchanged in this case.

Enter the Kähler calculus [3]. Underlied by Clifford algebra of differential forms – like tangent Clifford algebra underlies the geometric calculus– it brings about a fresh new view of quantum mechanics. This view arises, almost without effort, from the equation which is in this calculus what the Dirac equation is in traditional quantum mechanics. One does not need to first hypothesize foundations of quantum mechanics, which makes the Dirac version unintelligible even when one is adept at computing with it. Many foundations come in the wash from the mathematics and very little additional input. Not only Kähler theory reproduces the main Dirac-related results more elegantly, but does so far more profoundly and shows the way to further developments.

In this paper, we shall deal with differences between the Dirac and Kähler versions and, to a lesser extent, between Kähler and Hestenes. We limit ourselves to Kähler calculus of scalar-valued differential forms. That is all that one needs to supersede the Dirac and geometric calculus versions of relativistic quantum mechanics. We shall also give a very brief inkling of Kähler calculus with post-scalar-valued differential forms, as non-scalar-valuedness is needed for a unification of quantum and classical physics; the curvature and Einstein tensors are inadequate representations of what by their very nature are bivector-valued differential 2-forms and vector-valued differential 3-forms, respecively.

We shall show how the characteristics of the Kähler calculus bode well with the developments of some great ideas, proposed but not carried to fruition or accepted, by some geniuses of the 20$^{\text{th}}$ century, like Schwinger, Einstein, É. Cartan and Kähler, to name just the best known ones. We shall also be specific about gems, both mathematical and (mainly) physical, contained in this calculus. We shall also explain the mathematical philosophy of Kähler on a variety of issues (vector fields, differential forms, Lie differentiation, unification of derivatives, product of tangent algebras with algebra of integrands). Not suprisingly, his philosophy is the same as in É. Cartan's work. We shall also illustrate how some of the results that one achieves (mainly by Kähler himself) leave behind as not been sophisticated enough results which one finds deep into very specialized books on group theory, harmonic function theory, complex variable theory, cohomology theory, relativistic quantum mechanics and even particle theory.

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GENERELIZED LOCAL COHOMOLOGY OVER GRADED RINGS WITH SEMI-LOCAL BASE RING

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Let $R = \oplus_{j\geq 0} R_j$ be a homogeneous Noetherian ring with semi-local base ring R_0, i.e., R_0 has only finitely many maximal ideal. Let $R_+ = \oplus_{j\geq 1} R_j$ be the homogeneous ideal of R, generated by all positive degree homogeneous elements of R. We recall from [2] that a $\mathbb{Z}$-graded R-module T is tame or asymptotically gap free if T_n=0 for all $n \ll 0$, or else $T_n \neq 0$ for all $n \ll 0$. Recall also that, a sequence $(\mathscr{S}_n)_{n\in\mathbb{Z}}$ of subsets of $\mathrm{Spec}(R_0)$ is said to be *asymptotically stable* for $n \to -\infty$ if there exists $m \in \mathbb{Z}$ such that $\mathscr{S}_n = \mathscr{S}_m$ for all $n \leq m$. Using an idea of [2], for two finitely generated $\mathbb{Z}$-graded R-modules M and N, several results on the vanishing, Artinianness and tameness of the graded R-modules $H^i_{R_+}(M,N) = \varinjlim_{n\in\mathbb{N}} \mathrm{Ext}^i_R(M/(R_+)^n M, N)$ will be investigated.

Also, it will be shown that the sequence $\left(\mathrm{Ass}_{R_0}(H^i_{R_+}(M,N)_n)\right)_{n\in\mathbb{Z}}$ is asymptotically stable, which in turn, implies that the sequence $\left(\mathrm{Supp}_{R_0}(H^i_{R_+}(M,N)_n)\right)_{n\in\mathbb{Z}}$ is asymptotically stable too. Here, for an R_0-module X the symbols $\mathrm{Ass}_{R_0}(X)$ and $\mathrm{Supp}_{R_0}(X)$ stand for the set of all associated primes and support of X respectively [1].

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