In essence, the Kähler calculus is the extension of the Cartan calculus of differential forms that takes place when the underlying exterior algebra is replaced with Clifford algebra.

Kähler differentiation of differential forms α is here denoted by ∂ and defined as

$$\partial\alpha = d\alpha + \delta\alpha,$$

where

$$\partial\alpha \equiv dx^i \vee d_i\alpha = d\alpha + \delta\alpha$$

and where

$$d\alpha \equiv dx^i \wedge d_i\alpha, \qquad \delta\alpha = dx^i \cdot d_i\alpha$$

The terminology will be *Kähler*, *exterior*, *interior* and *covariant derivatives* or *differentials* for ∂, d, δ and d_i respectively. Be warned of the difference in terminology with the standard literature. Each type of product and of differentiation has its own Leibniz type of rule, and this dd is not zero in the most general cases.

We do not get into what $d_i\alpha$ is. For that, see chapters 1 and 3 in this web site. Suffice to say that, for scalar-valued differential forms, $d\alpha$ is the standard exterior derivative and $\delta\alpha$ is the co-derivative (or co-differential or divergence), as per a theorem of the Kähler calculus. $\delta\alpha$ depends on the metric through the Christoffel symbols, which, let us recall, are not zero even in Euclidean space if we do not use rectilinear coordinates. All differential operators now come together in a very profound way, including the Laplacian, $\partial\partial = (d+\delta)(d+\delta)$. But, all these formulas are now applicable to tensor-valued, exterior-valued and Clifford-valued differential forms, in which case all four terms in $\partial\partial$ (i.e. dd, $d\delta, ...$) may be different from zero.

The Kähler(-Dirac) equation pertains to the whole algebra, not only spinors. This equation supersedes Dirac's in all sorts of respects. The treatment of angular momentum, with *ab initio* inclusion of spin, is pure poetry. And forget about negative energy solutions; antiparticles emerge with the same sign as particles.

In the following we go in some more detail into the contents of this summary. At the moment of posting this, two chapters related to Kähler algebra and calculus shall have been posted.

Chapter 1 will not be presented in the summer school. It is for general orientation of those who know about other Clifford based calculi.

Chapter 2 will be for the first day of the second phase of the summer school (August 4). In the morning, we shall deal with the Clifford algebra of differential forms, which we shall call Kähler algebra. Late in the evening, we shall review and deal with two sections on applications, one of them to the computation of some real integrands that, in undergraduate courses, one usually computes with the help of the calculus of complex variable.

For explaining the contents of the second application, we here extend ourselves a little bit, since it will provide a perspective of how the whole summer course evolves around the following split of the unit:

$$1 = \varepsilon^+\tau^+ + \varepsilon^+\tau^- + \varepsilon^-\tau^+ + \varepsilon^-\tau^-, \tag{1}$$

where the $\varepsilon^\pm$ and $\tau^\pm$ are idempotents respectively connected with time translation and rotational symmetry, thus with mass and spin, or rest energy and rest angular momentum (If unfamiliar with the concept of idempotents, think simply of some type of special elements in the algebra). Kähler made the case that those four terms on the right of (1) represent electrons and positrons with both chirallities. We shall exploit over several chapters what this equation has to offer. In the last two chapters, we shall transcend this split.

Chapter 3 will deal with issues on differentiation That will be for the early morning of the second day of the second phase of the summer school (August 5). The applications will be for the evening and will comprise (a) Helmholtz theorem for scalar valued differential forms of arbitrary grade in Euclidean spaces of arbitrary dimension, (b) theory of real differential forms that mimics the calculus of complex variable, and not that just simplifies some real integrations, and (c) an improvement on Hodge's theorem.

Chapter 4 (August 6) will deal with the Kähler equation and how the Dirac equation is contained in it. Just from his equation, a variety of quantum mechanical issues emerge. Applications in relativistic quantum mechanics will be developed in the evening. The use of $\varepsilon^{\pm}\tau^{\pm}$ and the $\varepsilon^{\pm}\tau^{\mp}$ to represent electrons and positrons of both chirallities starts in this chapter.

Chapters 5 to 8 will be treated in the third phase of the summer school to the extent that time permits (August 8 and 9).

Chapter 5 will deal with symmetry and conservation. On the one hand there is the conservation law in exterior-interior calculus, which is what the Kähler calculus actually is. A uniqueness theorem that is required for the Helmholtz and Hodge's theorems immediately follows. A second application, already present in Kähler's work, is his derivation of the concept of charge through the split

$$u = {}^{+}u\,\varepsilon^{+} + {}^{-}u\,\varepsilon^{-}. \tag{2}$$

related to time translation and developed in an electromagnetic context. It comes in two types, positive and negative. At the same time, the particles and antiparticles represented on the right $\varepsilon^{\pm}\tau^{*}$ hand side of (1) pertain to the same sign of the energy.

Chapter 6 deals with angular momentum. Theory and applications here come together. By viewing angular momentum from a perspective of differential forms, not only do the orbital and spin parts come together, but actually are born together, not as twins, but as non-invariant terms which only become invariant after taking something from one of them and adding it to the other. Spin is as external as orbital, and orbital is as internal as spin. This confirms at a very profound level that the concept of particle emerges from the concept of field solutions of basic equations. The field is not exchange currency (quantized or not) among merchants (particles).

In chapter 7, we first start with the geometrization of the imaginary unit in quantum mechanics. The use of Clifford algebra in dealing with the phase factor replaces the imaginary unit with a bivector, and forces the same replacement in the idempotents as both of them, a phase factor and an idempotent, come together for each one-parameter symmetry. Three directions give rise to three generations. This raises the issue of which directions are those. This has an easy solution without renouncing the Lorentz transformations (key word: Reichenbach-Gruenbaum thesis of conventionality of synchronizations). The weak interaction, or something like it, clearly emerges. Kähler's Clifford algebra is replaced with the tensor product of two Clifford algebras. At the same time, one has extended the split (1), which was not possible before this geometrization.

Chapter 8 will deal with a second extension of the split due to the fact that the said tensor product contains a commutative substructure. Instead of having idempotents with just two factors, like the $\varepsilon^{\pm}\tau^{\pm}$ and the $\varepsilon^{\pm}\tau^{\mp}$, we shall now have three factors, the third one responding to space translation. But space translation is not like time translation or like rotational symmetry. This gives rise to very interesting issues like the use of these idempotents to represent quarks.

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