GENERELIZED LOCAL COHOMOLOGY OVER GRADED RINGS WITH SEMI-LOCAL BASE RING

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Let $R = \oplus_{j\geq 0} R_j$ be a homogeneous Noetherian ring with semi-local base ring R_0, i.e., R_0 has only finitely many maximal ideal. Let $R_+ = \oplus_{j\geq 1} R_j$ be the homogeneous ideal of R, generated by all positive degree homogeneous elements of R. We recall from [2] that a $\mathbb{Z}$-graded R-module T is tame or asymptotically gap free if T_n=0 for all $n \ll 0$, or else $T_n \neq 0$ for all $n \ll 0$. Recall also that, a sequence $(\mathscr{S}_n)_{n\in\mathbb{Z}}$ of subsets of $\mathrm{Spec}(R_0)$ is said to be *asymptotically stable* for $n \to -\infty$ if there exists $m \in \mathbb{Z}$ such that $\mathscr{S}_n = \mathscr{S}_m$ for all $n \leq m$. Using an idea of [2], for two finitely generated $\mathbb{Z}$-graded R-modules M and N, several results on the vanishing, Artinianness and tameness of the graded R-modules $H^i_{R_+}(M,N) = \varinjlim_{n\in\mathbb{N}} \mathrm{Ext}^i_R(M/(R_+)^n M, N)$ will be investigated. Also, it will be shown that the sequence $\big(\mathrm{Ass}_{R_0}(H^i_{R_+}(M,N)_n)\big)_{n\in\mathbb{Z}}$ is asymptotically stable, which in turn, implies that the sequence $\big(\mathrm{Supp}_{R_0}(H^i_{R_+}(M,N)_n)\big)_{n\in\mathbb{Z}}$ is asymptotically stable too. Here, for an R_0-module X the symbols $\mathrm{Ass}_{R_0}(X)$ and $\mathrm{Supp}_{R_0}(X)$ stand for the set of all associated primes and support of X respectively [1].

REFERENCES

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[2] M. Brodmann, M. Hellus, Cohomological patterns of coherent sheaves over projective schemes, J. Pure Applied Algebra, 2002, pp. 165-182.