

① a) $\frac{5}{\sqrt{12}} + \sqrt{27} = \frac{5}{2\sqrt{3}} + 3\sqrt{3} = \frac{5\sqrt{3}}{6} + 3\sqrt{3} = \frac{(5+18)\sqrt{3}}{6} = \boxed{\frac{23\sqrt{3}}{6}}$

b) $\frac{\sqrt[4]{x^3y^5}}{\sqrt{x^2}\sqrt{y}} = \sqrt[4]{\frac{x^3y^5}{x^4y}} = \sqrt[4]{\frac{y^4}{x}} = \frac{y}{\sqrt{x}} = \boxed{\frac{y\sqrt{x}}{x}}$

② a) $\sqrt{3-x} = \frac{9}{2} - 4x \Rightarrow 3-x = \frac{81}{4} + 16x^2 - 36x$
 $\Rightarrow 12-4x = 81 + 64x^2 - 144x \Rightarrow 64x^2 - 140x + 69 = 0$
 $\Rightarrow x = \frac{70 \pm \sqrt{4900 - 4416}}{64} = \frac{70 \pm 22}{64}$
 $\rightarrow \frac{92}{64} = \frac{23}{16}$ (faute)
 $\rightarrow \frac{48}{64} = \frac{3}{4}$ (bonne)

$x = \frac{3}{4}$

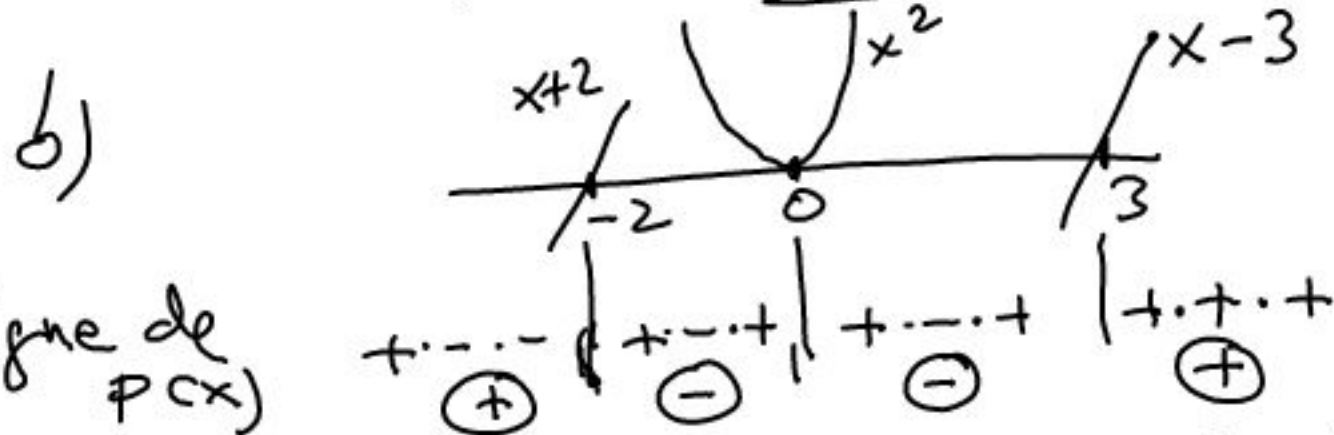
b) $\begin{cases} xy=1 \\ x^2+y^2=1 \end{cases} \Rightarrow x^2 - \frac{2}{x^2} = 1 \Rightarrow x^4 - x^2 - 2 = 0$
 $x^2 = \frac{1 \pm \sqrt{1+8}}{2} = \frac{1 \pm 3}{2} = \begin{matrix} 2 \\ -1 \end{matrix} \Rightarrow x = \pm\sqrt{2}$

$x = \sqrt{2} \Rightarrow y = \frac{\sqrt{2}}{2}$
 $y = -\sqrt{2} \Rightarrow y = -\frac{\sqrt{2}}{2}$

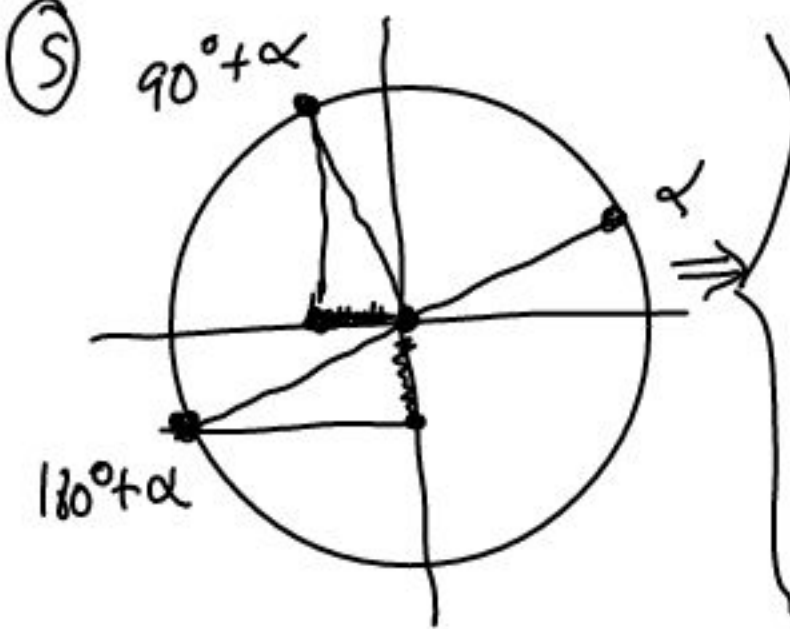
③ $P(x) = x^2(x^2 - x - 6) = 0 \Rightarrow \begin{cases} x=0 \\ x^2 - x - 6 = 0 \end{cases}$

a) $\Rightarrow \begin{cases} x=0 \\ x = \frac{1 \pm \sqrt{1+24}}{2} = \begin{matrix} 3 \\ -2 \end{matrix} \end{cases} \Rightarrow \text{racines: } \{0, 3, -2\}$

Decomposition: $x^2(x-3)(x+2)$

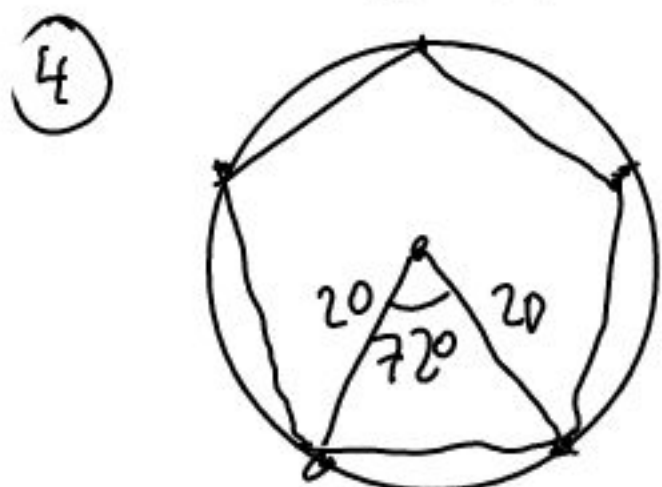


$P(x) > 0 \Rightarrow x \in (-\infty, -2) \cup (3, +\infty)$
 $P(x) < 0 \Rightarrow x \in (-2, 0) \cup (0, 3)$
 $P(x) = 0 \Rightarrow x \in \{0, 3, -2\}$



$$\begin{aligned} \tan(180^\circ + \alpha) &= \frac{\sin(180^\circ + \alpha)}{\cos(180^\circ + \alpha)} = \\ &= \frac{\cos(90^\circ + \alpha)}{-\sin(90^\circ + \alpha)} = \frac{-1/4}{-\sqrt{1 - (-1/4)^2}} = \frac{-1/4}{-\frac{\sqrt{15}}{4}} = \\ &= \boxed{\frac{1}{\sqrt{15}}} \approx 0.2582 \end{aligned}$$

$$\begin{aligned} \cos(90^\circ + \alpha) = -\frac{1}{4} &\Rightarrow 90^\circ + \alpha \approx 104.4775^\circ \Rightarrow \\ \Rightarrow 180^\circ + \alpha &= 194.4775^\circ \Rightarrow \tan(180^\circ + \alpha) \approx 0.2582 \end{aligned}$$



$$\begin{aligned} \frac{360^\circ}{5} &= 72^\circ \rightarrow \text{angle central} \\ \text{area} &= 5 \cdot \frac{1}{2} \cdot 20 \cdot 20 \cdot \sin 72^\circ = \\ &= 1000 \cdot \sin 72^\circ \approx 951.0565 \text{ cm}^2 \end{aligned}$$

⑥

$$\begin{aligned} 4 \cos(2x) + 3 \cos x &= 1 \\ 4 \cos^2 x - 4 \sin^2 x + 3 \cos x &= 1 \Leftrightarrow 8 \cos^2 x - 4 + 3 \cos x = 1 \\ \Leftrightarrow 8 \cos^2 x + 3 \cos x - 5 &= 0 \Rightarrow \cos x = \frac{-3 \pm \sqrt{9 + 160}}{16} \\ \Rightarrow \cos x = \frac{-3 \pm 13}{16} &= \frac{10}{16} = \frac{5}{8} \Rightarrow \end{aligned}$$

$$\begin{aligned} x &= 51^\circ 19' 4.13'' + m \cdot 360^\circ \\ x &= 180^\circ + m \cdot 360^\circ \end{aligned}$$

⑦ a) $\frac{2-i}{i^7} = \frac{2-i}{-i} \cdot \frac{i}{i} = \frac{2i - i^2}{-i^2} = \boxed{1+2i}$

b) $(2+3i)^3 = 8 + 3 \cdot 4 \cdot 3i + 3 \cdot 2 \cdot (3i)^2 + (3i)^3 =$
 $= 8 + 36i - 54 - 27i = \boxed{-46 + 9i}$