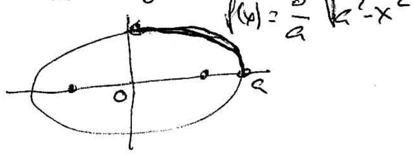
2BAT – 02/02/12

1) a) $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ $\quad f(x)=\frac{b}{a}\sqrt{a^2-x^2}$



$4\frac{b}{a}\int_0^a \sqrt{a^2-x^2}\,dx = 4\frac{b}{a}\int_0^{\pi/2} a\sqrt{1-\sin^2 t}\, a\cos t\,dt =$

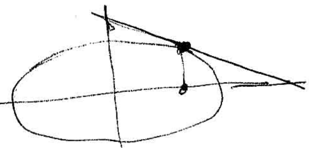
$x=a\sin t$
$dx=a\cos t\,dt$
$x=0 \rightarrow t=0$
$x=a \rightarrow t=\frac{\pi}{2}$

$=\frac{4b}{a}a^2\int_0^{\pi/2}\cos^2 t\,dt =$
$=4ab\int_0^{\pi/2}\frac{1+\cos 2t}{2}\,dt =$
$=4ab\left[\frac{t}{2}+\frac{\sin(2t)}{4}\right]_0^{\pi/2}=$

$=4ab\left[\left(\frac{\pi}{4}+0\right)-(0+0)\right]= \quad 4ab\cdot\frac{\pi}{4}=\pi ab$

b) $a\simeq 6{,}6$
$b\simeq 4{,}2$ $\rightarrow \simeq \pi\cdot 3{,}3\cdot 10\cdot 2{,}1\cdot 10\ m^2 = \boxed{2177{,}12\ m^2}$

c)



$\frac{2x}{a^2}+\frac{2yy'}{b^2}=0 \Rightarrow y'=-\frac{b^2\cdot \cancel{2}x}{a^2\cdot\cancel{2}y}=-\frac{b^2}{a^2}\frac{x}{\frac{b}{a}\sqrt{a^2-x^2}}=-\frac{b}{a}\frac{x}{\sqrt{a^2-x^2}}$

$y'(c)=-\frac{b}{a}\frac{c}{\sqrt{a^2-c^2}}=-\frac{\cancel{b}}{a}\frac{c}{\cancel{b}}=-\frac{c}{a}$

$y-\frac{b}{a}\sqrt{a^2-c^2}=-\frac{c}{a}(x-c) \Rightarrow y-\frac{b^2}{a}=-\frac{c}{a}x+\frac{c^2}{a}$

$ay+cx=b^2+c^2$ $\Rightarrow \boxed{ay+cx=a^2}$

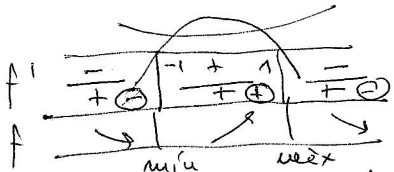
2) a) f contínua en $[a,b]$ i derivable en $(a,b) \Rightarrow \exists\alpha\in(a,b)$ tal que $f'(\alpha)=\frac{f(b)-f(a)}{b-a}$

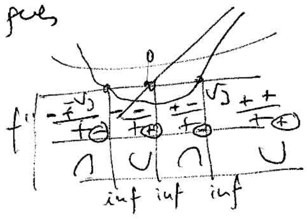
$\frac{f(x)-f(0)}{x-0}=f'(\alpha) \Longleftrightarrow \frac{\arctan x}{x}=\frac{1}{1+\alpha^2} \Longleftrightarrow \arctan x=\frac{x}{1+\alpha^2}>\frac{x}{1+x^2}$

$(0<\alpha<x)$ $\quad (0<\alpha<x)$ $\quad \left(\begin{array}{c}0<1+\alpha^2<1+x^2\\ \frac{1}{1+\alpha^2}>\frac{1}{1+x^2}\end{array}\right)$

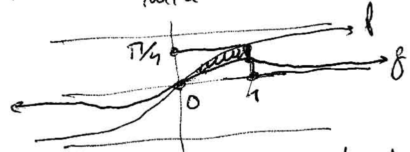
b) AH $\lim_{x\to\infty}\frac{x}{1+x^2}=\lim_{x\to\infty}\frac{\frac{1}{x}}{\frac{1}{x^2}+1}=\frac{0}{0+1}=0 \Rightarrow \boxed{y=0}$

No hi ha verticals ni obliqües





c)



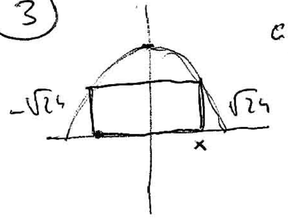
$\int_0^1\left(\arctan x-\frac{x}{1+x^2}\right)dx=$

$\left[x\arctan x\right]_0^1-\int_0^1\frac{x}{1+x^2}dx-\int_0^1\frac{x}{1+x^2}dx$

be' $\begin{cases} u=\arctan x & du=\frac{1}{1+x^2}dx \\ dv=dx & v=x\end{cases}$

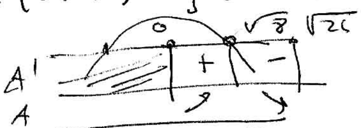
$=(\arctan 1-\arctan 0)-\int_0^1\frac{2x}{1+x^2}dx=$
$=\frac{\pi}{4}-0-\left[\ln(1+x^2)\right]_0^1=\boxed{\frac{\pi}{4}-\ln 2}\simeq 0{,}09225$

3)



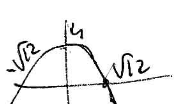
a) $A(x)=2x\left(4-\frac{x^2}{6}\right)=\frac{1}{3}x(24-x^2)=\frac{1}{3}(24x-x^3)$, $0<x<\sqrt{24}$

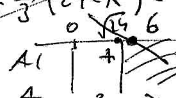
$A'(x)=\frac{1}{3}(24-3x^2)$



àrea màxima
base: $2\sqrt{8}$
alçada: $4-\frac{8}{6}=4-\frac{4}{3}=\frac{8}{3}$ $\quad \boxed{\text{àrea}=\frac{16\sqrt{8}}{3}=\frac{32\sqrt{2}}{3}}\simeq 15.0849$

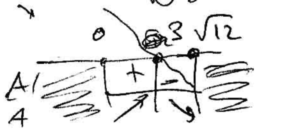
b) $P(x)=4x+2\left(4-\frac{x^2}{6}\right)=4x+\frac{1}{3}(24-x^2)=-\frac{1}{3}x^2+4x+8$ $\quad 0<x<\sqrt{24}$
$P'(x)=-\frac{2}{3}x+4$





La funció creix en $(0,\sqrt{24})$
No hi ha màxim en $(0,\sqrt{24})$

c) $P(x)=4x+2\left(4-\frac{x^2}{3}\right)=-\frac{2}{3}x^2+4x+8$
$P'(x)=-\frac{4}{3}x+4$



Màxim per a $x=3$
base $=6$
alçada $=1$ $\quad \boxed{\text{perímetre}=14}$