17/11/2011 – 2 BAT

① a) $S_1 = 1^3$, $S_1 = (1)^2$ iguals per a $n=1$

Suposem $1^3+2^3+\dots+n^3=(1+2+\dots+n)^2$

Cal demostrar-lo per a $n+1$:

$$1^3+2^3+\dots+n^3+(n+1)^3=(1+2+\dots+n)^2+(n+1)^3=\left[\frac{n(n+1)}{2}\right]^2+(n+1)^3=$$

$$=(n+1)^2\left(\frac{n^2}{4}+n+1\right)=(n+1)^2\frac{(n+2)^2}{4}=\left[\frac{(n+1)(n+2)}{2}\right]^2=\left[1+2+\dots+n+(n+1)\right]^2$$

b) $$\lim_{n\to\infty}\frac{1^3+\dots+n^3}{n^4}=\lim_{n\to\infty}\frac{n^2(n+1)^2}{4n^4}=\lim_{n\to\infty}\frac{n^4+2n^3+n^2}{4n^4}=\lim_{n\to\infty}\frac{1+2\frac{1}{n}+\frac{1}{n^2}}{4}=\frac{1+0+0}{4}=\boxed{\frac{1}{4}}$$

② a) $\log\frac{8x^3}{3^3}=\log(2x+2) \Longrightarrow 8x^3=54x+54 \Longrightarrow 4x^3-27x-27=0$

Busquem arrels amb la regla de Ruffini:

3	4	0	−27	−27
		12	36	27
	4	12	9	0

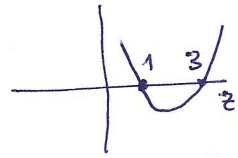
$$x=\frac{-6\pm\sqrt{36-36}}{4}=-\frac{3}{2}$$

$\Rightarrow$ $x=3$; $\cancel{x=-\frac{3}{2}}$ ← No és solució perquè $\log\left(8\cdot\left(-\frac{3}{2}\right)^3\right)$ no existeix

b) $4^x-4\cdot 2^x+3<0 \Rightarrow (2^x)^2-4\cdot 2^x+3<0$

Fem $2^x=z$: $z^2-4z+3<0$

$z^2-4z+3=0 \Leftrightarrow z=1$ o $z=3$



$\Rightarrow 1<z<3$

$\Rightarrow 1<2^x<3 \Rightarrow \log 1 < x\log 2 < \log 3 \Rightarrow \frac{0}{\log 2}<x<\frac{\log 3}{\log 2}$

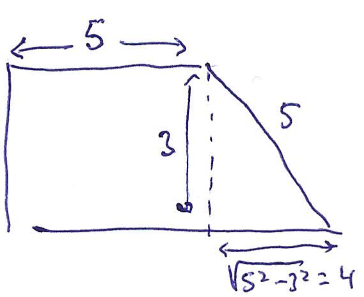
$$\boxed{x\in\left(0,\frac{\log 3}{\log 2}\right)}$$

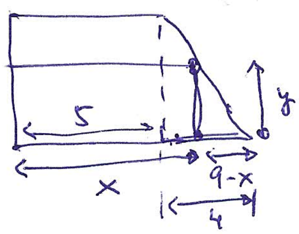
③ a) $$\lim_{x\to 2^+}(1+x)^{\frac{1}{2-x}}=3^{\frac{1}{0^-}}=3^{-\infty}=\frac{1}{3^{+\infty}}=\frac{1}{+\infty}=\boxed{0}$$

b) $$\lim_{n\to\infty}\left(\frac{1+n}{n}\right)^{n^2}=\left(1^{+\infty}\right)$$ Indeterminat

$$=\lim_{n\to\infty}\left[\left(1+\frac{1}{n}\right)^n\right]^2=\left(\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n\right)^2=\boxed{e^2}$$

④

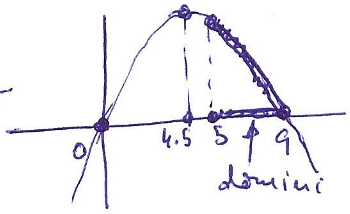




$\frac{9-x}{y}=\frac{4}{3} \Rightarrow y=\frac{3}{4}(9-x)$

Àrea $=x\cdot y=\frac{3}{4}(9x-x^2)$, $5<x<9$

↓ gràfic



El màxim es troba quan $x=5$ i el seu valor és $\frac{3}{4}(45-25)=15$

⑤ $f(x)=\begin{cases} x+e^{\frac{x}{1+x}} & , \text{ si } x>-1 \\ -1 & , \text{ si } x=-1 \end{cases}$

a) $\lim_{x\to -1^+} f(x) = -1+e^{\frac{-1}{0^+}} = -1+e^{-\infty} = -1+\frac{1}{e^{+\infty}} = -1+\frac{1}{+\infty} = -1+0 = \boxed{-1}$

b) La funció és contínua en $[-1,0]$ (l'únic punt dubtós seria $x=-1$, però hem vist que $\lim_{x\to -1^+} f(x) = f(-1)$).

Observem que $\left.\begin{array}{l} f(-1)=-1 \\ f(0)=0+e^0=1 \\ f \text{ cont. en } [-1,0] \end{array}\right\} \xrightarrow{\text{T. Bolzano}}$ existeix $x\in(-1,0)\subset[-1,0]$ en què $f(x)=0$.

c) $\boxed{y=mx+b \text{ A.Ob}}$
$m=\lim_{x\to +\infty} \frac{x+e^{\frac{x}{1+x}}}{x} = \lim_{x\to+\infty} 1+\frac{e^{\frac{x}{1+x}}}{x} = 1+\frac{e^{\frac{1}{0+1}}}{+\infty} = 1+\frac{e}{+\infty} = 1+0 = 1$

$b=\lim_{x\to+\infty} x+e^{\frac{x}{1+x}} - 1\cdot x = \lim_{x\to+\infty} e^{\frac{x}{1+x}} = e^{\frac{1}{0+1}} = e^1 = \boxed{e}$

$\Rightarrow \boxed{y=x+e}$

⑥ $f(x)=\frac{x^2-2x-1}{1-x}$ Talls eix OX: $x^2-2x-1=0 \Rightarrow x=1\pm\sqrt{5}$
Tall OY: $f(0)=\frac{-1}{1}=-1$

A.V.: $\lim_{x\to 1^\pm} f(x) = \frac{-2}{0^\pm} = \pm\infty \Rightarrow \boxed{x=1 \text{ A.V}}$

A.H.: $\lim_{x\to+\infty} \frac{x^2-2x-1}{1-x} = \lim \frac{1-\frac{2}{x}-\frac{1}{x^2}}{\frac{1}{x^2}-\frac{1}{x}} = \frac{1}{0} = \infty \rightarrow$ No existeix

A.Ob: $m=\lim_{x\to\pm\infty} \frac{f(x)}{x} = \lim \frac{x^2-2x-1}{x-x^2} = \lim_{x\to\pm\infty} \frac{1-\frac{2}{x}-\frac{1}{x^2}}{\frac{1}{x}-1} = \frac{1}{-1} = \boxed{-1}$

$b=\lim_{x\to\pm\infty}(f(x)+x) = \lim \frac{x^2-2x-1-x+x^2}{1-x} = \lim_{x\to\pm\infty} \frac{-3x-1}{1-x} = \frac{-3-0}{0-1} = \boxed{3}$

$\Rightarrow \boxed{y=-x+3}$

⑦ $\left.\begin{array}{l} a_1=3 \\ a_n=K\cdot a_{n-1}, \ n>1 \end{array}\right\} \Rightarrow \left\{\begin{array}{l} S_{21}=\frac{a_1(K^{21}-1)}{K-1} \\ \frac{a_{21}}{a_1}=K^{20} \Rightarrow \frac{115.0128}{3}=38,3376=K^{20} \Rightarrow K=1.2 \end{array}\right\} \Rightarrow$

$\Rightarrow S_{21} = \frac{3(1.2^{21}-1)}{0,2} \approx \boxed{675.0767986}$

$= \boxed{15(1.2^{21}-1)}$