2BAT - 9/5/16

(1)
$$\underbrace{\underbrace{\begin{pmatrix} 1 & -5 & -8 \\ 3 & -1 & -a \\ 1 & a & 3 \end{pmatrix}}_{A}\left| \begin{matrix} 1 \\ 3 \\ 1 \end{matrix}\right.}_{A/B}$$

$\det A = -3+5a-24a-8+a^2+45 = a^2-19a+34$

$\det A = 0 \Leftrightarrow a = \frac{19\pm\sqrt{361-136}}{2} = \frac{19\pm15}{2} = 17,\ 2$

$\det A \neq 0 \Leftrightarrow a\neq 17$ i $a \neq 2$

1) $\boxed{a\neq 17 \text{ i } a\neq 2}$ $\det A\neq 0 \Rightarrow r(A)=3=r(A/B) \Rightarrow$ sistema compatible determinat

2) $\boxed{a=2}$ $M=\begin{vmatrix} 1 & -5 \\ 3 & -1\end{vmatrix} = -1+15=14\neq 0$, orla $\rightarrow \begin{vmatrix} 1 & -5 & 1 \\ 3 & -1 & 3 \\ 1 & 2 & 1\end{vmatrix} =$
$\det A = 0$

$\Rightarrow r(A)=2$ i $r(A/B)=2 \Rightarrow$ sistema compatible indeterminat

~~sistema comt~~

3) $\boxed{a=17}$ $M\neq 0$, $\det A\neq 0$, orla $\begin{vmatrix} 1 & -5 & 1 \\ 3 & -1 & 3 \\ 1 & 17 & 1\end{vmatrix} =$

$\Rightarrow r(A)=2=r(A/B) \Rightarrow$ sistema compatible indeterminat

1) Els tres plans tenen un únic punt en comú.
2) i 3) Els tres plans es tallen en una recta.

(2) $r\begin{cases} x-1=-y+2 \\ x-1=\frac{z-1}{3}\end{cases} \Rightarrow$ Feix de plans que contenen r: $x+y-3+\alpha(3x-z-2)=0$

a) Punt mitjà de AB: $M=\left(\frac{1-3}{2}, \frac{4+2}{2}, \frac{2+0}{2}\right)=(-1,3,1)$

Pla que conté r i M: $-1+3-3+\alpha(-3-\cancel{1}-2)=0 \Rightarrow \alpha=\frac{1}{-6}$

$x+y-3-\frac{1}{6}(3x-z-2)=0 \Leftrightarrow \boxed{3x+6y+z-16=0}$

b) $\pi: \begin{vmatrix} x-1 & 1 & 1 \\ y & 1 & 0 \\ z-2 & -1 & -2\end{vmatrix} = -2(x-1)+y-(z-2)=0 \Leftrightarrow 2x-y+z-4=0 \Rightarrow$

$\Rightarrow$ vector perpendicular a $\pi = (2,-1,1)$ }
vector director de $r = (1,-1,3)$ } si α és l'angle buscat,

tenim $\sin\alpha = \frac{|(2,-1,1)\cdot(1,-1,3)|}{\sqrt{4+1+1}\sqrt{1+1+9}} = \frac{|2+1+3|}{\sqrt6\sqrt{11}} = \frac{6}{\sqrt6\sqrt{11}} = \sqrt{\frac{6}{11}} \Rightarrow$

$\alpha = 47°36'28.64''$

(3A) existeix $A^{-1} \Leftrightarrow \det A\neq 0 \Leftrightarrow \begin{vmatrix} a & 1 & 1 \\ 1 & a & 1 \\ 1 & 1 & a\end{vmatrix}\neq 0 \Leftrightarrow a^3+1+1-a-a-a\neq 0$

$\Leftrightarrow a^3-3a+2\neq 0$

$a^3-3a+2=0 \xrightarrow{\text{Regla de Ruffini}}$

	1	0	-3	2
1		1	1	-2
	1	1	-2	0

$a=\frac{-1\pm\sqrt{1+8}}{2} = 1,\ -2$

$\det A\neq 0 \Leftrightarrow \boxed{a\neq 1 \text{ i } a\neq -2 \Leftrightarrow \text{existeix } A^{-1}}$

3B

$$A = \begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix} \quad B = \begin{pmatrix} 3 & 0 \\ 1 & 2 \end{pmatrix} \quad C = \begin{pmatrix} 1 & -3 \\ 0 & 4 \end{pmatrix}$$

$$AX + B = C \iff AX = C - B \iff A^{-1}AX = A^{-1}(C-B) \iff X = A^{-1}(C-B)$$

$$A^{-1}: \left(\begin{array}{cc|cc} 1 & -1 & 1 & 0 \\ 2 & 3 & 0 & 1 \end{array}\right) \underset{1F_2 - 2F_1 \to F_2}{\iff} \left(\begin{array}{cc|cc} 1 & -1 & 1 & 0 \\ 0 & 5 & -2 & 1 \end{array}\right) \underset{5F_1 + F_2 \to F_1}{\iff} \left(\begin{array}{cc|cc} 5 & 0 & 3 & 1 \\ 0 & 5 & -2 & 1 \end{array}\right) \underset{\substack{F_1/5 \to F_1 \\ F_2/5 \to F_2}}{\iff} \underbrace{\left(\begin{array}{cc|cc} 1 & 0 & 3/5 & 1/5 \\ 0 & 1 & -2/5 & 1/5 \end{array}\right)}_{A^{-1}}$$

$$X = \underbrace{\begin{pmatrix} 3/5 & 1/5 \\ -2/5 & 1/5 \end{pmatrix}}_{A^{-1}} \underbrace{\begin{pmatrix} -2 & -3 \\ -1 & 2 \end{pmatrix}}_{C-B} = \begin{pmatrix} -\frac{6}{5} - \frac{1}{5} & -\frac{9}{5} + \frac{2}{5} \\ \frac{4}{5} - \frac{1}{5} & \frac{6}{5} + \frac{2}{5} \end{pmatrix} = \boxed{\begin{pmatrix} -\frac{7}{5} & -\frac{7}{5} \\ \frac{3}{5} & \frac{8}{5} \end{pmatrix}}$$