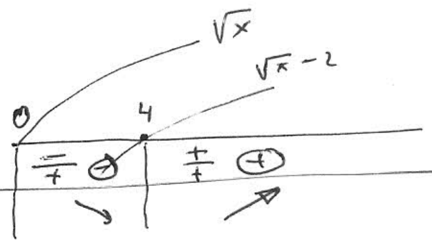
2BAT 19/5/16



(C1) $f(x) = x - 4\sqrt{x}$

a) $f'(x) = 1 - \frac{4}{2\sqrt{x}} = 1 - \frac{2}{\sqrt{x}} = \frac{\sqrt{x}-2}{\sqrt{x}}$

f monòtona decreixent en (0,4)

f " " en (4,+∞)

Màxim local: (0,0)

Mínim local i absolut: (4, f(4)) = (4, −4)

b) La recta és del tipus $y - f(a) = f'(a)(x-a)$

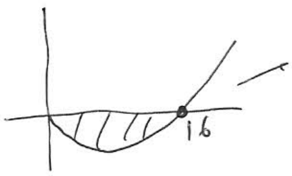
En ser paral·lela a $y = \frac{1}{3}x$, s'obté que $f'(a) = \frac{1}{3} \Rightarrow 1 - \frac{2}{\sqrt{a}} = \frac{1}{3}$

$\Rightarrow \frac{2}{\sqrt{a}} = 1 - \frac{1}{3} = \frac{2}{3} \Rightarrow \sqrt{a} = 3 \Rightarrow a = 9$

Pertant, la solució és

$y - f(9) = \frac{1}{3}(x-9) \Leftrightarrow y-(-3) = \frac{1}{3}(x-9) \Leftrightarrow 3y+9 = x-9 \Leftrightarrow \boxed{x - 3y - 18 = 0}$

c)



$x - 4\sqrt{x} = 0 \Leftrightarrow \sqrt{x}(\sqrt{x}-4) = 0 \Leftrightarrow x=0,\ x=16$

$\text{Àrea} = -\int_0^{16}(x-4\sqrt{x})\,dx = -\left(\frac{x^2}{2} - \frac{4x^{3/2}}{3/2}\right]_0^{16} = -\left(128 - \frac{4\cdot 64}{3/2}\right) =$

$= -\left(128 - \frac{8\cdot 64}{3}\right) = \frac{-384+512}{3} = \boxed{+\frac{128}{3}} \simeq 42{,}667$

(C2) a) $p(x) = x^4 + ax^3 + bx^2 + 3x$

a = −2, b = −4 ?

$0 = p(3) = 81 + 27a + 9b + 9 \Leftrightarrow 3a + b = -10$

$p'(x) = 4x^3 + 3ax^2 + 2bx + 3$

~~$-5 = p'(2) = 32 + 12a + 4b + 3 \Leftrightarrow 12a + 4b = -40$~~

$1 = p'(-1) = -4 + 3a - 2b + 3 \Leftrightarrow 3a - 2b = 2$

$\begin{cases} 3a + b = -10 \\ 3a - 2b = 2 \end{cases}$ $\quad 3b = -12 \quad \boxed{b = -4,\ a = -2}$ Arrels

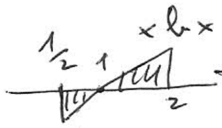
	1	−2	−4	3
3		3	3	−3
	1	1	−1	0

$x = \frac{-1 \pm \sqrt{1+4}}{2} = \frac{-1 \pm \sqrt{5}}{2}$

$p'(x) = 4x^3 - 6x^2 - 8x + 3$ $\quad p'(-1) = 1$

$[x = 3,\ x = 0,\ x = \frac{-1+\sqrt{5}}{2},\ x = \frac{-1-\sqrt{5}}{2}]$ $\quad -4 - 6 + 8 + 3$

b)



$\text{Àrea} = -\int_{1/2}^{1} x\ln x\,dx + \int_1^2 x \ln x\,dx$

$\int x\ln x = \frac{x^2}{2}\ln x - \int \frac{x}{2}dx = \frac{x^2}{2}\ln x - \frac{x^2}{4} = \frac{x^2}{4}(2\ln x - 1)$

$\ln x = u \rightarrow u' = \frac{1}{x}dx$

$x\,dx = dv \rightarrow v = \int x\,dx = \frac{x^2}{2}$

$\text{Àrea} = -\left(\frac{1}{4}(-1) - \frac{1}{16}(-2\ln 2 - 1)\right) + \left(2\ln 2 - 1 - \frac{1}{4}(-1)\right)$

$\text{Àrea} = \frac{1}{4} - \frac{1}{8}\ln 2 - \frac{1}{16} + 2\ln 2 - 1 + \frac{1}{4} = \boxed{\frac{15}{8}\ln 2 - \frac{9}{16}} \simeq 0{,}73715$

(C3) a) $\text{Preu}(x) = P(x) = \text{Volum}(x) \cdot \text{Preu m}^3(x) = \pi(20+x)^2(800+40x)(40-x)\cdot 10^{-6}$

b) $x \geq 0$

1 €/m³

1 €/10⁶

48000

$P(x) = 40\cdot 10^{-6}\pi(20+x)^3(40-x)$

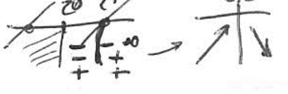
$P(x) = 4\pi\cdot 10^{-5}(20+x)^3(40-x) = 4\pi 10^{-5}(8000 + 1200x + 60x^2 + x^3)(40-x)$

$P(x) = 4\pi 10^{-5}(-x^4 - 20x^3 + 1200x^2 + 40000x + 320000)$

$P'(x) = 4\pi 10^{-5}(-4x^3 - 60x^2 + 2400x + 40000) = -16\pi 10^{-5}(x-25)(x+20)^2$

$P'(x) = -16\pi 10^{-5}(x^3 + 15x^2 - 600x - 10000)$

x = 25 màxim



c)

	1	20	−1200	−40000	−320000
40		40	2400	48000	320000
	1	60	1200	8000	0

$\to P(40)=0 \to$ 40 anys

$P(x)=\dots(40-x)=0$
$x=40$

$\det A = 6+140-27-36+7-90=0$

A1

a) $\begin{matrix}\vec e_1\\ \vec e_2\\ \vec e_3\end{matrix}\begin{pmatrix}1&10&9\\3&2&7\\2&-1&3\end{pmatrix} \Leftrightarrow \begin{matrix}\vec e_1\\ \vec e_2-3\vec e_1\\ \vec e_3-2\vec e_1\end{matrix}\begin{pmatrix}1&10&9\\0&-28&-20\\0&-21&-15\end{pmatrix} \Leftrightarrow \begin{matrix}\vec e_1\\ \vec e_2-3\vec e_1\\ 4(\vec e_3-2\vec e_1)-3(\vec e_2-3\vec e_1)\end{matrix}\begin{pmatrix}1&10&9\\0&-28&-20\\0&0&0\end{pmatrix}$

Relació de dependència: $\boxed{\vec e_1-3\vec e_2+4\vec e_3=0}$

$\boxed{\text{rang}=2}$ perquè equival al rang de $\begin{pmatrix}1&10&9\\0&-28&20\end{pmatrix}$

b) $\begin{pmatrix}1&10&9\\3&2&7\\2&-1&3\end{pmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}0\\0\\0\end{pmatrix} \Leftrightarrow \begin{pmatrix}1&10&9\\0&-7&-5\end{pmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}0\\0\end{pmatrix}$

$z=\lambda \quad \begin{cases}x+10y=-9\lambda\\ -7y=-5\lambda\end{cases} \Rightarrow \begin{cases}z=\lambda\\ y=\frac{5}{7}\lambda\\ x=-9\lambda+\frac{50}{7}\lambda=\frac{-13}{7}\lambda\end{cases}$

$\frac{\lambda}{7}=\alpha$

$\boxed{\alpha(13,5,-7)}$ $\quad \boxed{\begin{pmatrix}13\alpha\\5\alpha\\-7\alpha\end{pmatrix},\ \alpha\in\mathbb{R}}$

A2

$\begin{pmatrix}a&1&-3&a\\1&1&2&4\\3&1&-4a&0\end{pmatrix}$ (M, M′)

$\det A=-4a^2+6-3+9-2a+4a$
$\det A=-4a^2+2a+12 \begin{cases}=0\\ \neq 0\end{cases}$
$-2a^2+a+6=0 \Leftarrow a=\frac{-1\pm\sqrt{1+48}}{-4}= \begin{cases}-3/2\\ 2\end{cases}$

$\boxed{a\neq 2 \text{ i } a\neq -\frac{3}{2}} \Leftrightarrow r(A)=r(A|B)=3 \Leftrightarrow$ S.C.D. $\Leftrightarrow$ Tres plans secants en un punt

$\boxed{a=-3/2}$ $\begin{vmatrix}1&-3&-3/2\\1&2&4\\1&6&0\end{vmatrix}=0-12-9+3+24-0=6\neq 0$ $\left\{\begin{matrix}r(A)=2\\ \Rightarrow r(A|B)=3\end{matrix}\right.$ S.I. Els plans

i $M\neq 0$ i $\det A=0$

$\boxed{a=2}$ $\begin{vmatrix}1&-3&2\\1&2&4\\1&-8&0\end{vmatrix}=0-12-16-4-0+32=0$ $\left\{\begin{matrix}r(A)=2=r(A|B) \quad \text{S.C.I.}\\ \text{Els plans es tallen en una recta i són diferents}\end{matrix}\right.$

i $M\neq 0$ i $\det A=0$

Equació: $\begin{pmatrix}2&1&-3&2\\1&1&2&4\end{pmatrix}$ $\begin{cases}2x+y=2+3\lambda\\ x+y=4-2\lambda\end{cases}$

$z=\lambda$
$x=\frac{\begin{vmatrix}2+3\lambda&1\\4-2\lambda&1\end{vmatrix}}{1}=-2+5\lambda$
$y=\frac{\begin{vmatrix}2&2+3\lambda\\1&4-2\lambda\end{vmatrix}}{1}=6-7\lambda$

$\boxed{\frac{y-6}{-7}=\frac{x+2}{5}=\frac{z}{1}}$

A3 $r:\ x=y=2z$

(a) $\pi_\alpha: x-y+\alpha(y-2z)=0$ (feix de plans que conté r) $\Longleftrightarrow x+(\alpha-1)y-2\alpha z=0$

$A(3,-1,2) \quad 3=d(A,\pi_\alpha)=\frac{|1\cdot 3+(\alpha-1)(-1)-2\alpha\cdot 2|}{\sqrt{1+(\alpha-1)^2+(-2\alpha)^2}}=\frac{|-5\alpha+4|}{\sqrt{5\alpha^2-2\alpha+2}} \Rightarrow$

$\Rightarrow 9(5\alpha^2-2\alpha+2)=25\alpha^2-40\alpha+16 \Rightarrow 20\alpha^2+22\alpha+2=0 \Rightarrow 10\alpha^2+11\alpha+1=0$

$\alpha=\frac{-11\pm\sqrt{121-40}}{20}=\frac{-11\pm 9}{20}= \begin{cases}-1/10\\ -1\end{cases} \Rightarrow$

$\pi_1: x-\frac{11}{10}y+\frac{2}{10}z=0$
$\pi_2: x-2y+2z=0$
$\pi_1: 10x-11y+2z=0$

(b)

$2=d[(2\lambda,3\lambda,\lambda),\ 3x+4y-12z-5]=\frac{|6\lambda+8\lambda-12\lambda-5|}{13}=\frac{|2\lambda-5|}{13}$

$26=2\lambda-5$ o $-26=2\lambda-5 \Leftarrow \begin{cases}\lambda=\frac{-21}{2}\\ \lambda=\frac{+31}{2}\end{cases} \Rightarrow$

Punts:
$\left(\frac{-21}{1},\frac{-21}{1},\frac{-21}{2}\right)$
$\left(\frac{+31}{1},\frac{+31}{1},\frac{-31}{2}\right)$