Matemàtiques 2n BAT - 30/11/15

1) $f(x) = \begin{cases} \frac{5}{x-1}, & x<2 \\ 9-x^2, & x\geq 2 \end{cases}$

a) $f(1) = \frac{5}{1-1} = \frac{5}{0}$ no existeix

$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} \frac{5}{x-1} = \frac{5}{0^+} = +\infty$

$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} \frac{5}{x-1} = \frac{5}{0^-} = -\infty$

$\Rightarrow$ Discontinuïtat asimptòtica en $x=1$

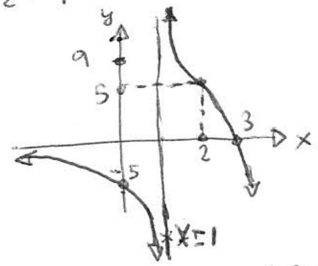
$f(2) = 9-2^2 = 5$

$\lim_{x\to 2^-} f(x) = \frac{5}{2-1} = 5$

$\lim_{x\to 2^+} f(x) = 9-2^2 = 5$

$\Rightarrow$ f es contínua en $x=2$

b)



$y = \frac{5}{x-1}$ → A.V.: $x=1$; → A.H.: $y=0$; → Tall OY: $(0,-5)$

$y = 9-x^2$ → Talls OX $(3,0)$, $(-3,0)$; → Màxim: $(0,9)$

2) a) $I(0) = 12000e^{-0{,}2\cdot 0} = \boxed{12000 \text{ infectes}}$

b) $0{,}6\cdot 12000 = I(t) = 12000\cdot e^{-0.2t} \Rightarrow 0{,}6 = e^{-0.2t} \Rightarrow \ln(0.6) = -0.2t \underbrace{\ln e}_{1}$

$\Rightarrow t = \frac{\ln(0.6)}{-0.2} \simeq \boxed{2{,}55 \text{ setmanes}}$

c) $1 = I(t) = 12000\cdot e^{-0.2t} \Rightarrow \frac{1}{12000} = e^{-0.2t} \Rightarrow -\ln 12000 = -0.2t\cdot \underbrace{\ln e}_{1}$

$\Rightarrow t = \frac{\ln 12000}{0.2} \simeq \boxed{46.96 \text{ setmanes}}$

3) $f(x) = -x^3 + ax^2 + bx - 12$

$13 + f(1) - 33 = 0 \Rightarrow f(1) = 20 \Rightarrow 20 = -1+a+b-12$

$f'(1) = -13 \Rightarrow \ldots \Rightarrow -13 = -3+2a+b$

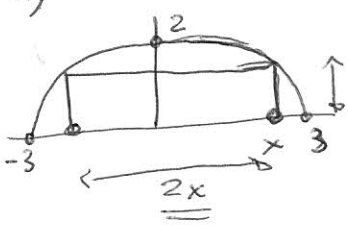
$f'(x) = -3x^2 + 2ax + b$

$\begin{cases} a+b = 33 \\ 2a+b = -10 \end{cases}$ $\begin{cases} a+b = 33 \\ a = -43 \end{cases}$ $\Rightarrow b = 76$

$\boxed{\begin{array}{l} a=-43 \\ b=76 \end{array}}$

4) $\frac{x^2}{9} + \frac{y^2}{4} = 1$

a)



$y^2 = 4\left(1-\frac{x^2}{9}\right) \Rightarrow y = \frac{2}{3}\sqrt{9-x^2}$

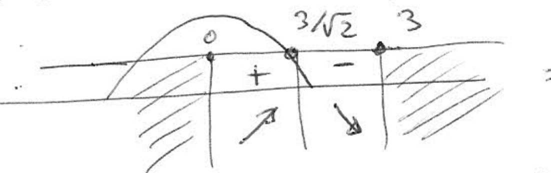
Àrea: $A(x) = 2x\cdot\frac{2}{3}\sqrt{9-x^2}$

$\Rightarrow \boxed{A(x) = \frac{4}{3}x\sqrt{9-x^2}, \quad 0<x<3}$

b) $A'(x) = \frac{4}{3}\left(\sqrt{9-x^2} + x\frac{(-2x)}{2\sqrt{9-x^2}}\right) = \frac{4}{3}\left(\frac{9-x^2-x^2}{\sqrt{9-x^2}}\right) = \frac{4}{3}\cdot\frac{9-2x^2}{\sqrt{9-x^2}}$

Signe(A')

Monotonia(A)



$\Rightarrow$ $A'(3/\sqrt{2}) = 0$

A creix en $(0, \frac{3}{\sqrt{2}})$ $\Rightarrow$

A decreix en $(\frac{3}{\sqrt{2}}, 0)$

$\Rightarrow$ A presenta un màxim local i absolut en $x = \frac{3}{\sqrt{2}}$ i

l'àrea màxima és $A\left(\frac{3}{\sqrt{2}}\right) = \frac{4}{3}\cdot\frac{3}{\sqrt{2}}\sqrt{9-\frac{9}{2}} = \frac{4}{\sqrt{2}}\cdot\frac{3}{\sqrt{2}} = \frac{12}{2} = \boxed{6\ u^2}$

5.

$f(x) = x^2 \cdot e^x \Rightarrow Dom(f) = \mathbb{R}$

$f'(x) = 2x \cdot e^x + x^2 e^x = e^x(x^2+2x)$

$f''(x) = (2x+2)e^x + (x^2+2x)e^x = e^x(x^2+4x+2)$

$\Leftarrow \boxed{e^x > 0, \forall x \in \mathbb{R}}$

$f'(x) = 0 \Rightarrow x = 0, x = -2$

$f''(x) = 0 \Rightarrow x = \frac{-2 \pm \sqrt{4-2}}{1} = -2+\sqrt{2}, \ -2-\sqrt{2}$

x^2+2x

		-2		0	
Signe (f')	$+ \cdot + \ (+)$		$+ \cdot - \ (-)$		$+ \cdot + \ (+)$
Monot (f)	↗		↘		↗

x^2+4x+2

		$-2-\sqrt{2}$		$-2+\sqrt{2}$	
signe f''	$+ \cdot + \ (+)$		$+ \cdot - \ (-)$		$+ \cdot + \ (+)$
Concav (f)	∪		∩		∪

$f(0) = 0$

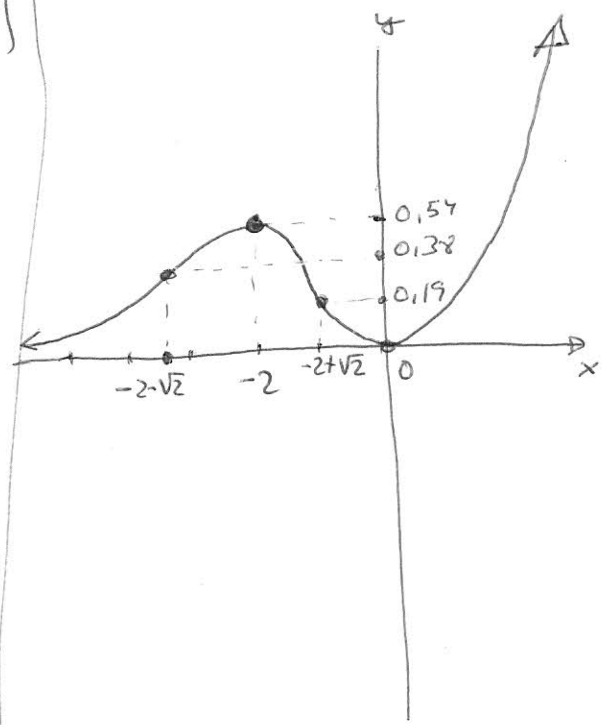
$f(-2) = 4 \cdot e^{-2} = \frac{4}{e^2} \simeq 0{,}54$

$f(-2+\sqrt{2}) \simeq 0.19$

$f(-2-\sqrt{2}) \simeq 0{,}38$

Creix en $(-\infty, -2) \cup (0, +\infty)$
Decreix en $(-2, 0)$
Còncava amunt en $(-\infty, -2-\sqrt{2}) \cup (-2+\sqrt{2}, +\infty)$
Còncava avall en $(-2-\sqrt{2}, -2+\sqrt{2})$

Gràfic



A.H.

$\lim_{x \to +\infty} x^2 e^x = (+\infty)\cdot(+\infty) = +\infty$

$\lim_{x \to -\infty} x^2 e^x = +\infty \cdot e^{-\infty} = +\infty \cdot 0 = \text{Indet} = \lim_{x \to -\infty} \frac{x^2}{e^{-x}} \overset{(R.H.)}{=} \lim_{x \to -\infty} \frac{2x}{-e^{-x}} = \left(\frac{-\infty}{-\infty}\right)$

$(R.H.) = \lim_{x \to -\infty} \frac{2}{e^{-x}} = \frac{2}{e^{+\infty}} = \frac{2}{+\infty} = 0 \Rightarrow$

$\Rightarrow \boxed{y = 0 \text{ és A.H.}}$

A.V.

$\lim_{x \to a} x^2 e^x = a^2 \cdot e^a \in \mathbb{R}, \Rightarrow$

$\Rightarrow \boxed{\text{no existeix A.V.}}$

A.Ob.

$\lim_{x \to +\infty} \frac{x^2 e^x}{x} = \lim_{x \to +\infty} x \cdot e^x = +\infty$

$\lim_{x \to -\infty} \frac{x^2 e^x}{x} = \lim_{x \to -\infty} x e^x = \lim_{x \to -\infty} \frac{x}{e^{-x}} = \frac{-\infty}{+\infty}$

$(R.H.) = \lim_{x \to -\infty} \frac{1}{-e^{-x}} = \frac{1}{-\infty} = 0 \Rightarrow$

$\Rightarrow$ No hi ha Asímptota Oblíqua

6.

$f(x) = x \ln x$

$f'(x) = \ln x + x \cdot \frac{1}{x} = \ln x + 1$

$f''(x) = \frac{1}{x} = \frac{(-1)^2 (2-2)!}{x^{2-1}}$

$f^{(k+1)}(x) = \left(f^{(k)}\right)'(x) = \left[\frac{(-1)^k (k-2)!}{x^{k-1}}\right]' = (-1)^k (k-2)! \left(x^{-k+1}\right)' = (-1)^k (k-2)! (-k+1) x^{-k} =$

$= \frac{(-1)^k (k-2)! (k-1)(-1)}{x^k} = \boxed{\frac{(-1)^{k+1}(k-1)!}{x^k}}$