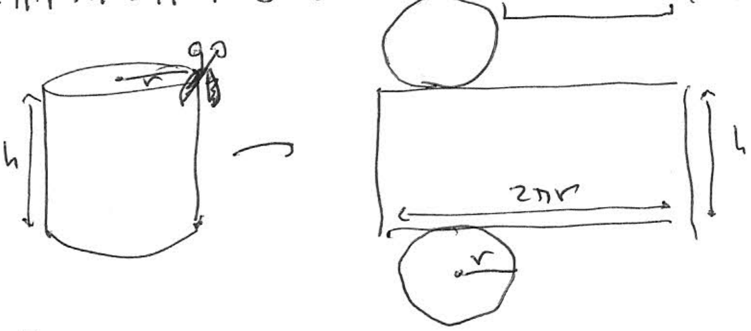
4 ESO A 29/02/16

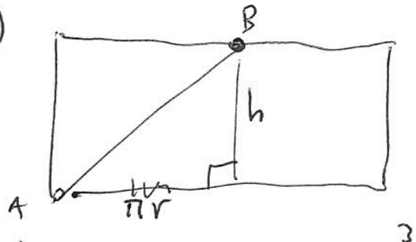
1. a) $V = \pi \cdot r^2 \cdot h = \pi \cdot 1^2 \cdot 3 = 3\pi\ m^3 \simeq \boxed{9{,}425\ m^3}\ (9{,}424777961) \simeq \underline{9425\ l}$

b)



c) $S = 2\pi r \cdot h = 2\pi \cdot 1 \cdot 3 = 6\pi\ m^2 \simeq 18{,}84955592\ m^2 \simeq \boxed{18{,}850\ m^2}$

d)



$AB = \sqrt{\pi^2 r^2 + h^2} = \sqrt{\pi^2 + 9} \simeq \boxed{4{,}344\ m}$

e) $x^3 = 3\pi \Rightarrow x = \sqrt[3]{3\pi} \simeq \boxed{2{,}112\ m.}$

2. a) $9x^4 + 9x^2 + 2 = 0$ $\boxed{\text{no té solució}}$ perquè $9x^4 + 9x^2 + 2 \geq 2 > 0,\ \forall x$

b) $(x-4)(x+4) = x + 14 \Leftrightarrow x^2 - 16 = x + 14 \Leftrightarrow x^2 - x - 30 = 0 \Leftrightarrow x = \frac{1 \pm \sqrt{1+120}}{2} = \frac{1 \pm 11}{2} = \boxed{6,\ -5}$

c) $3x - \sqrt{1-x} = \frac{7}{4} \Leftrightarrow -\sqrt{1-x} = \frac{7}{4} - 3x \Leftrightarrow 4\sqrt{1-x} = -7 + 12x \Rightarrow$

$\Rightarrow 16(1-x) = 49 + 144x^2 - 168x \Rightarrow 144x^2 - 152x + 33 = 0$

$$x = \frac{152 \pm \sqrt{152^2 - 19008}}{288} = \frac{152 \pm \sqrt{4096}}{288} = \frac{152 \pm 64}{288} = \begin{cases} \frac{216}{288} = \frac{3}{4} \\ \frac{88}{288} = \frac{11}{36} \end{cases}$$

Comprovació: $x = \frac{3}{4} \Rightarrow 3 \cdot \frac{3}{4} - \sqrt{1 - \frac{3}{4}} = \frac{9}{4} - \frac{1}{2} = \frac{7}{4} \Rightarrow \boxed{x = \frac{3}{4} \text{ és solució}}$

$x = \frac{11}{36} \Rightarrow 3 \cdot \frac{11}{36} - \sqrt{1 - \frac{11}{36}} = \frac{11}{12} - \sqrt{\frac{25}{36}} = \frac{11}{12} - \frac{5}{6} = \frac{11-10}{12} = \frac{1}{12} \neq \frac{7}{4} \Rightarrow$

$\Rightarrow x = \frac{11}{36}$ $\underline{\underline{\text{no}}}$ és solució

3. a) Cal trobar x tal que $\sqrt[5]{x} = -2$. Llavors, $x = (-2)^5 = \boxed{-32}$

b) x = costat $\Rightarrow x^3 = 0{,}008\ m^3 \Rightarrow x = \sqrt[3]{0{,}008} = \boxed{0{,}2\ m = 20\ cm}$

c) $12\ hm^3 = 12 \cdot 10^6\ m^3$ $\Rightarrow$ nombre de piscines $= \frac{12 \cdot 10^6}{2500} = \frac{12 \cdot 10^4}{25} = \frac{12 \cdot 10^{4-2} \cdot 4}{100} = \boxed{4800 \text{ piscines}}$

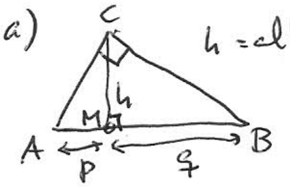
Volum piscina $= 50 \cdot 25 \cdot 2 = 2500\ m^3$

2. d) $\begin{cases} x - 2y = 10 \\ 3x + 3y = 21 \end{cases} \Leftrightarrow \begin{cases} x - 2y = 10 \\ x + y = 7 \end{cases} \Leftrightarrow \begin{cases} x - 2y = 10 \\ -3y = 3 \end{cases} \Leftrightarrow \begin{matrix} x = 10 + 2(-1) = 8 \\ y = -1 \end{matrix}$ $\boxed{\begin{matrix} x = 8 \\ y = -1 \end{matrix}}$

e) $\frac{5x}{20} - \frac{x-5}{6} = \frac{x}{15} + 1 \overset{(\cdot 60)}{\Leftrightarrow} 15x - 10(x-5) = 4x + 60 \Leftrightarrow 5x + 50 = 4x + 60$

$\Leftrightarrow x = 60 - 50 \Leftrightarrow \boxed{x = 10}$

④ a)

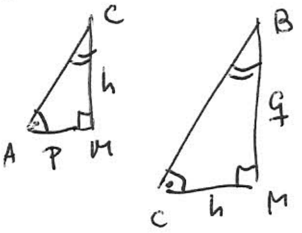


h = altura sobre la hipotenusa

p i q ↔ projeccions dels catets sobre la hipotenusa

$\left.\begin{array}{l}\widehat{ACM}+\widehat{MCB}=90^\circ\\ \widehat{ACM}+\widehat{MAC}=90^\circ\end{array}\right\} \Rightarrow \widehat{MCB}=\widehat{MAC}$ i, per tant,

$\widehat{MBC}=90^\circ-\widehat{MCB}=90^\circ-\widehat{MAC}=\widehat{ACM}$. És a dir tenim dos triangles amb els angles iguals



Llavors, els costats són proporcionals

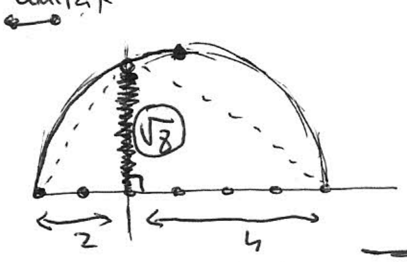
$\boxed{\dfrac{p}{h}=\dfrac{h}{q}}$ q.e.d

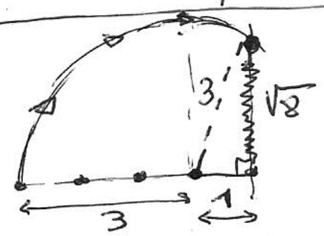
(quod erat demonstrandum)

(la qual cosa volíem demostrar)

b) $\left.\begin{array}{l}\dfrac{p}{h}=\dfrac{h}{q} \Rightarrow h^2=pq \Rightarrow h=\sqrt{pq}\\ \sqrt{8}=\sqrt{4\cdot 2}\end{array}\right\} \Rightarrow$ Només cal construir l'altura sobre la hipotenusa d'un triangle rectangle d'hipotenusa $h=4+2=6$ i projeccions dels catets sobre aquesta, $p=4$ i $q=2$.

unitat $\Leftarrow$





c) A partir del teorema de Pitàgores i d'observar que $\sqrt{8}=\sqrt{9-1}=\sqrt{3^2-1^2}$. D'aquí surt que si construïm un triangle rectangle d'hipotenusa = 3 i un catet = 1, llavors l'altre catet mesura $\sqrt{8}$.

⑤ a) $\sqrt{6}\,\sqrt{12}\,\sqrt{2}=\sqrt{6\cdot 12\cdot 2}=\sqrt{144}=\sqrt{72\cdot 2}=\sqrt{36\cdot 4}=\sqrt{12\cdot 12}=\sqrt{12^2}=\boxed{12}$

b) $\dfrac{\sqrt{15}}{\sqrt[3]{15}}=\sqrt[6]{\dfrac{15^3}{15^2}}=\boxed{\sqrt[6]{15}}$

c) $2\sqrt{8}+\sqrt{32}=2\cdot\sqrt{2^2\cdot 2}+\sqrt{2^4\cdot 2}=2\cdot 2\sqrt{2}+2^2\sqrt{2}=4\sqrt{2}+4\sqrt{2}=\boxed{8\sqrt{2}}$

d) $\sqrt{ab^3}\cdot\sqrt[4]{b^2\cdot a}=\sqrt[4]{a^2b^6\cdot b^2a}=\sqrt[4]{a^3b^8}=\boxed{b^2\cdot\sqrt[4]{a^3}}$